13 DESIGN OF BEAM-COLUMNS - I

1.0 INTRODUCTION

Columns in practice rarely experience concentric axial compression alone. Since columns are usually parts of a frame, they experience both bending moment and axial force. The frames, in which columns are members, may be classified as *braced* or *unbraced.* In braced frames the resistance to lateral loads at floor levels is provided either by bracings [Fig. 1(*b*)] or shear walls. In case of unbraced frames [Fig. 1(*d*)] the resistance to lateral loads is obtained from the members of the frames with moment resisting connections between them. Thus the relative translation between the ends of a column in a braced frame is prevented, whereas in unbraced frames the columns are free to sway causing relative translation between their ends. More details on classification of frames as braced and unbraced are given in the chapter on frames. Thus columns in practice experience bending about one or both axis in addition to axial compression, due to one or more of the following reasons.

- The compressive force may be eccentrically transferred to the column [Fig. 1(*a*)]. When this eccentric force is transferred to the centre line of the column, an equivalent axial compression and bending moment act on the column.

- When the beams in braced rigid portal frames are subjected to gravity loads, the rotation of the beams at their junction with the column causes rotation of the column also at the junction due to rigid connection [Fig. 1(*b*)]. Hence beam transfers bending moments to the column in addition to axial load [Fig. 1(*c*)].

- When a multi-storey multi-bay un-braced frame is subjected to gravity loads and lateral loads due to wind or earthquake, the columns are subjected to sway deflection and bending [Fig 1(*d*)]. In such cases, the columns experience axial compression as well as bending moments [Fig.1 (*e*)].

- Beams may frame from two orthogonal directions in corner columns in buildings [Fig. 1(*f*)]. In such cases the columns may be subjected to bending about both principal axes in addition to axial compression [Fig. 1(*f*)].

Columns subjected to combined axial force and bending moment are referred to as beam-columns. A beam-column may be subjected to single curvature bending over its length [Fig. 1(*c*)]. In this case the nature of the bending stress (compressive or tensile) at a point in the cross section and sign of the bending moment diagram over its entire length of the beam-column remains the same. Consequently, the curvature has the same sign over the entire length of the column. On the other hand, the columns in a sway frame [Fig. 1(*d*)] experience reverse curvature bending as shown in Fig. 1(*e*), causing variation of the nature (positive or negative) of the bending moment and curvature over the length of the column.

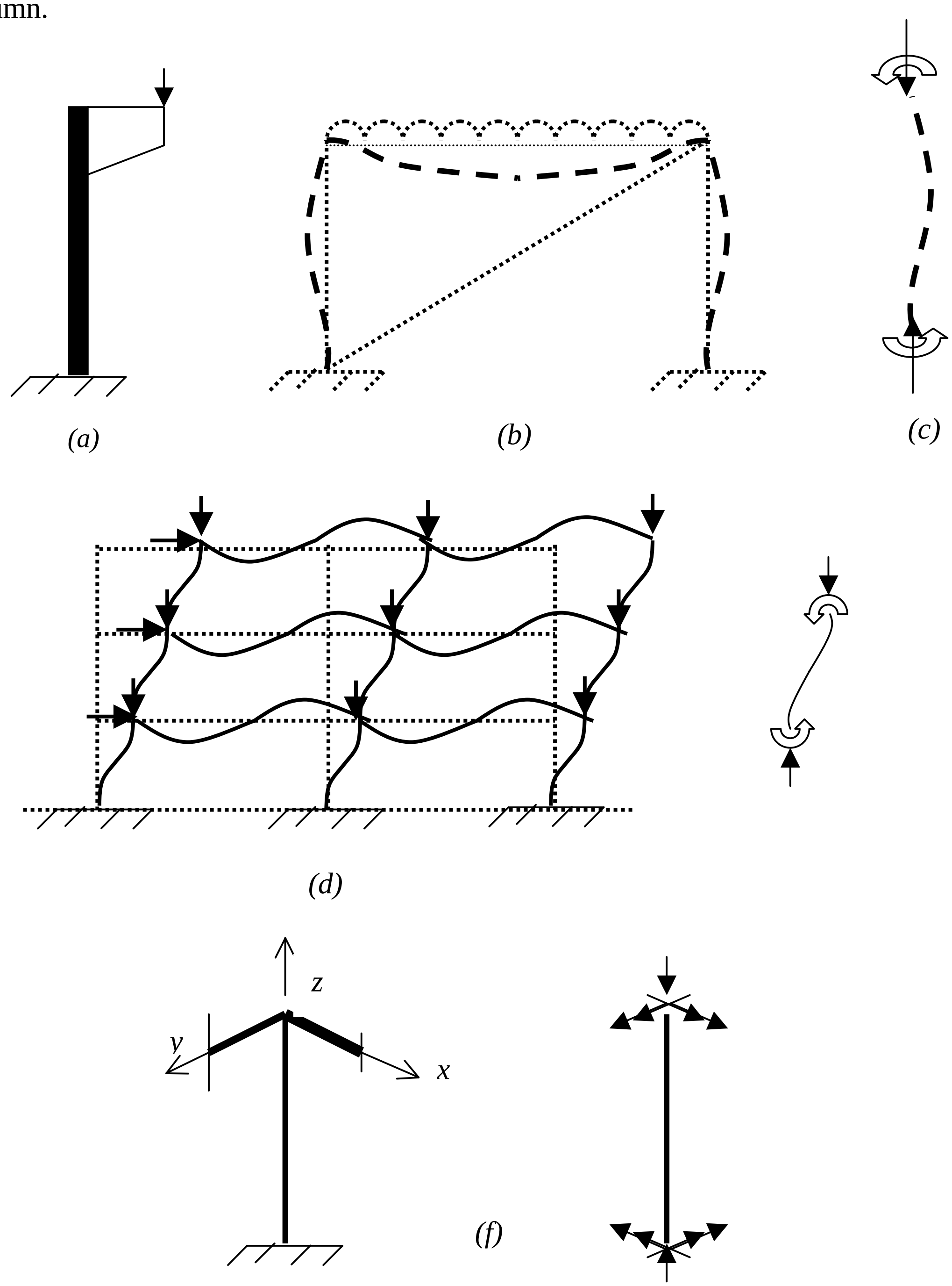


Fig. 1 Beam-Columns in Frames

Presence of bending moments in the beam-columns reduces the axial force at which they fail. This topic presented in two parts, deals with the behaviour, and design of beam-columns. In Part I initially, the behaviour and strength of short beam-column members under combined compression and bending moment are discussed. In such short beam column the failure is due to the strength of the material being reached (material failure). Subsequently, the behaviour and strength of practical, long beam-columns, as affected by stability and deformation, are discussed. In long columns the failure may be either due to material strength being reached at the ends of the column or instability of the overall column. In Part II equations for the design of beam-columns subjected to combination of axial compression and biaxial bending are presented. A design example of a beam-column also is presented in Part II.

2.0 SHORT BEAM-COLUMNS

A short member (stub column), made of non-slender (plastic, compact or semi-compact) section under axial compression, fails by yielding (due to large deformation) at the squash load, P_d, given by [Fig. 2(*a*)]

$$P_d = A_g f_y \quad (1)$$

where, f_y is the yield strength of the material, and A_g is the gross area of the cross section.

If the stub column is made of *slender cross section*, the plate elements of the cross section undergo local buckling before reaching the yield stress. This causes reduction in the effective area of the cross section to a value below the gross area, A_g, and the member fails at a load below P_d, given by Eqn.1.

Similarly a short member made of plastic or compact section and subjected to only bending moment fails at the plastic moment capacity, M_p, given by [Fig. 2(*b*)]

$$M_p = Z_p . f_y \quad (2)$$

where, Z_p = plastic section modulus of the cross section, in the case of plastic and compact sections.

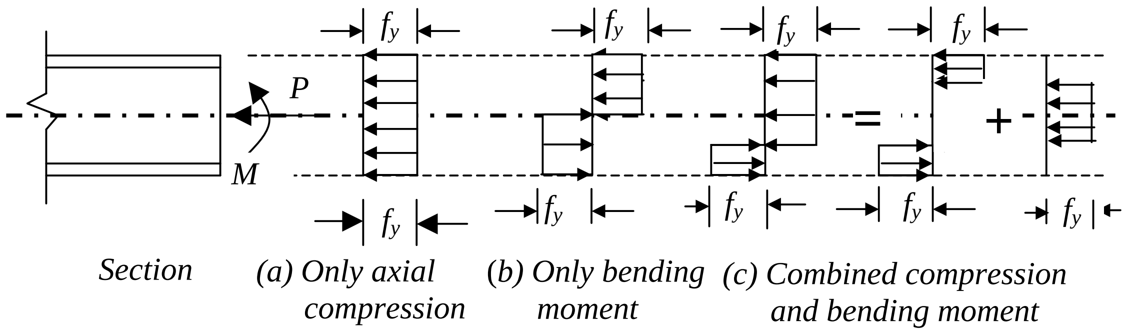


Fig. 2 Stresses in Short Beam-Columns

A semi-compact section subjected to bending moment only fails by buckling of a plate element of the cross section before the plastification of the entire section as shown in Fig. 2 (*b*) but after the stress at the extreme fibre in compression reaches the yield stress. In a slender section, the plate elements buckle even before the extreme fibre stress in bending reaches the yield stress. Consequently, the semi-compact and slender sections fail under bending even before reaching the plastic moment, M_p, given by Eqn. 2.

The discussions that follow generally assume that the cross section is either plastic or compact. In the case of slender and semi-compact sections the effect of earlier failure before complete section yielding has to be considered. The strength of such members may be analysed by following the procedure discussed in the chapters on *cold-formed steel members.*

The stress distribution at failure over the (plastic or compact) cross-section of a beam-column under combined compression and bending moment is shown in Fig. 1(*c*). It can be modeled as superposition of only compressive stress over an area of the cross section close to the neutral axis of the cross section and the balance of the section subjected to compressive and tensile stress due to bending. Hence such beam-columns fail before reaching the squash load, P_d, given by equation 1 or the plastic moment, M_p, given by Eqn. 2. The typical failure envelope diagram of a stub beam-column made of I section and subjected to axial compression P and bending moment M is shown in Fig. 3, in a non-dimensional form. At smaller values of axial compression, only a small area of the cross section closer to the neutral axis is necessary to equilibrate the external compression, P. Since the area closer to the neutral axis contributes very little to the plastic moment capacity, M_p, of the cross section, the reduction in the moment capacity, M, is negligible when the axial compression is small. It is seen in the failure envelope that for smaller axial compression ($P / P_d < 0.15$) the reduction in the moment capacity is negligible ($M/M_p \approx 1.0$)

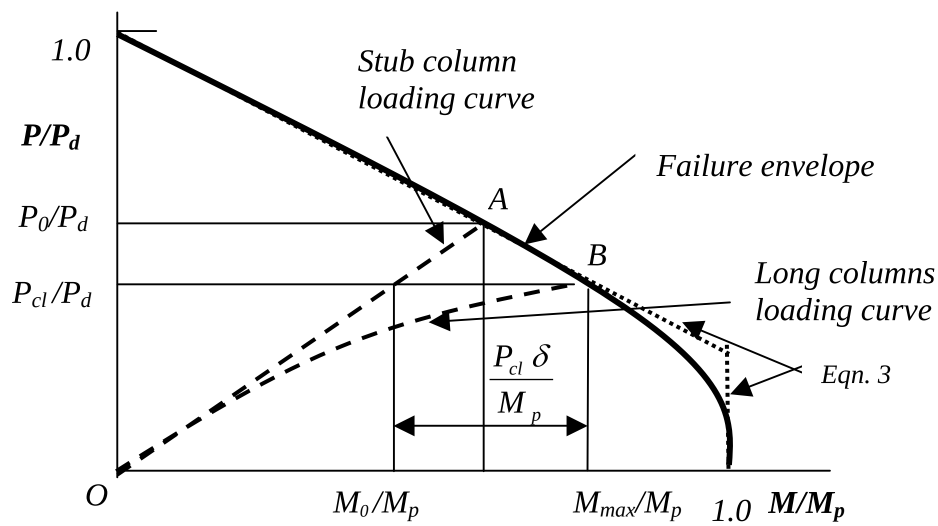


Fig. 3 Beam-Column Failure Envelope

Eqn. 3 in a non-dimensional form gives the failure envelope under the major axis bending and the axial compression, as shown in Fig. 3.

$$\frac{P}{P_d} + \frac{0.85M}{M_p} \leq 1.0$$
$$\frac{M}{M_p} \qquad \leq 1.0 \qquad (3)$$

Although there is a small reduction in the bending moment at lower values of axial compression as seen in Fig. 3, in Eqn. 3 this has been disregarded. The interaction equation (Eqn. 3) is linear up to the plastic moment capacity, M_p, of the member.

The loading curve of a short beam-column by a compressive force at a constant eccentricity, is indicated by a straight line, such as OA (Fig. 3) having a constant slope. The slope is dictated by the eccentricity. When this loading path OA intersects the failure envelope curve at A, the beam-column strength is reached. The values of P and M corresponding to point A are the compression and moment capacity of short the beam-column under the given eccentricity. This may be calculated from Eqn. 3 by substituting $P.e$ for M, where e is the eccentricity of the compressive force, P.

3.0 LONG BEAM-COLUMNS

Typically steel columns in practice are long and slender. Such slender columns when axially compressed tend to fail by buckling rather than yielding, as discussed in the chapter on *Introduction to Column Buckling*. Similarly, slender I sections subjected to bending moment about the major axis (z-axis) when not laterally supported, may fail by lateral-torsional buckling, as discussed earlier in the chapter on *Unrestrained Beam Design*. However, under minor axis (y-axis) bending, the plastic and compact sections will reach the plastic moment capacity, M_{py}, without undergoing premature lateral buckling or local buckling. Such long slender members subjected to combined axial compression and bending may experience different modes of instability or material failure. These are discussed in this section.

Consider a slender beam-column subjected only to equal and opposite end moment, M_o, as shown in Fig. 4(*a*). The beam-column is bent into a single curvature with a maximum deflection δ_0, as shown by the dotted line in Fig. 4(*a*). If the axial compression is applied at the ends of the column now, additional bending moment is caused due to the axial load acting on the deformed shape. This additional bending moment causes additional deflection and so on, until the final maximum deflection δ is reached at the stage of equilibrium under combined axial force and bending moments. This is referred to as P-δ effects. The final deflected shape and the final bending moment diagram, considering the P-δ effect, are shown by dashed curves in Fig. 4(*a*). It is seen that due to P-δ effect, the maximum moment in the beam-column, M_{max}, is larger than the externally applied end moments, M_o.

The same beam-column, when subjected to equal end moments acting in the same direction, experiences double curvature bending in addition to axial compression as shown in Fig. 4(*b*). The deflected shape as well as the bending moment diagram of the beam column, not considering P-δ effects, are shown in Fig. 4(*b*) by dotted curves and after considering the P-δ effects is indicated by the dashed curve. It is seen that although δ is greater than δ_o, in this case, the magnified moment considering the P-δ effects need not be greater than the end moments M_o. Thus, it is seen that the P-δ effects and the magnified moments depend upon the moment gradient over the length of the member. The discussion so far was about beam-columns in frames braced against sway.

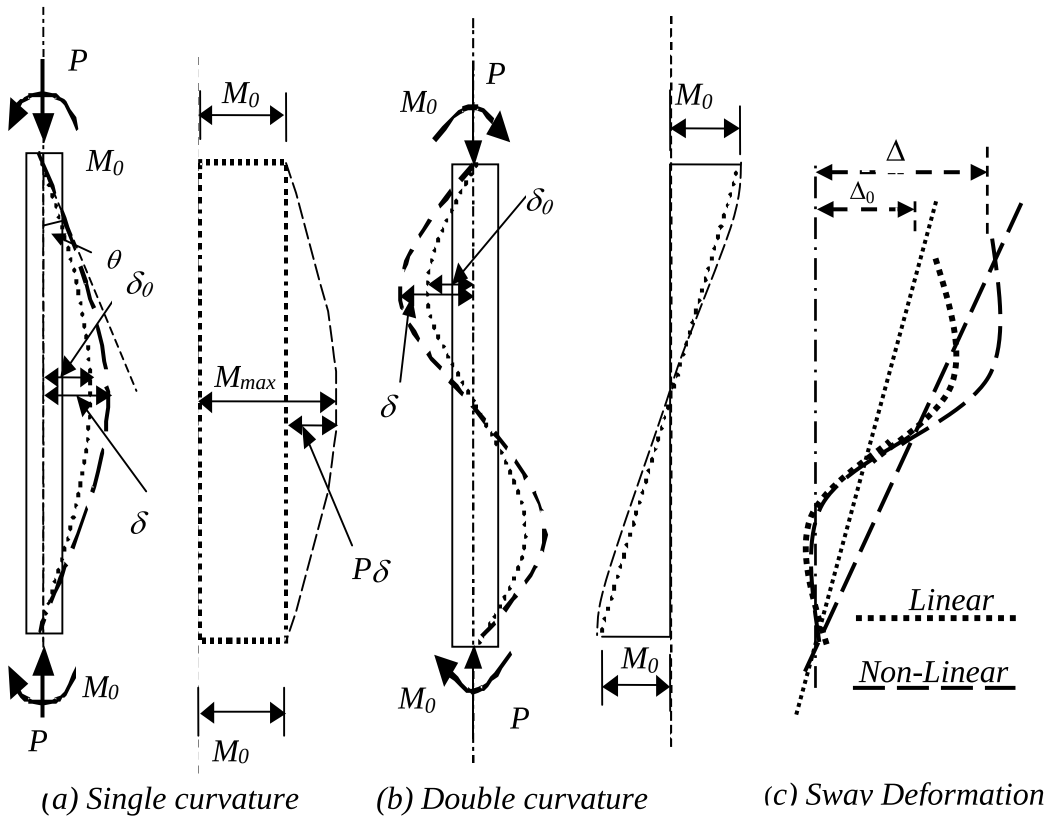
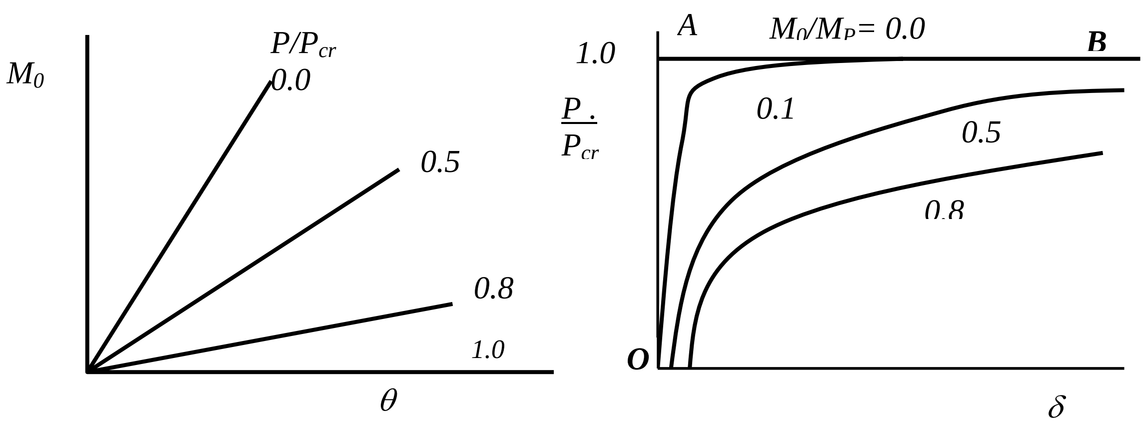


Fig. 4 Deflection and Moment Magnification

If a frame is not braced to prevent lateral sway, linear elastic analysis for lateral loads may indicate that column ends translate relative to one another by a distance Δ_0, in addition to end rotations and reverse curvature deformation as indicated by dotted curve in Fig. 4(*c*). The axial forces, P, acting on the frame with sway displacement, Δ_0, increase the sway to Δ, as shown by dashed curve and the column and beam moments also increase. This additional displacement can be obtained only by a non-linear analysis of the frame considering the equilibrium of the frame in the deformed configuration. This increase in sway deformation and bending moments, due to the load acting on the deformed structure, are referred to as P-Δ effects.

Let us see the load-deformation behaviour of a beam-column, when the axial compression and end moments are applied one after another. If an axial compression, P, is initially applied so that P/P_{cr} is maintained constant while the moments at the two ends of the beam-column are proportionately increased in the elastic range of the material, the moment-end rotation diagram is linear as shown by Fig. 5(*a*). However, it is seen that the stiffness of the beam-column (the slope of the moment-rotation line) decreases with increase in the initially applied axial compression. When the $P/P_{cr} = 1.0$, the end rotation, θ [Fig. 4(*a*)] increases to infinity even for $M=0$, indicating instability under axial compression itself.

On the other hand, if the ratios of the initially applied end moments, M_0/M_p, is maintained constant and the axial compression alone is increased, the compressive load versus lateral deformation behaviour of the beam-column in the elastic range is as per Fig. 5(*b*). In this case the load deformation behaviour is non-linear. It is seen that a perfect column subjected to axial compression without any end moments undergoes bifurcation type of buckling [OAB in Fig.5 (*b*)]. If some end moments are applied initially, the member undergoes initial deflection, δ_o, the magnitude of which depends upon the magnitude of the end moments. Subsequently, as the axial compression is increased gradually, the lateral deflection increases even from the very beginning. Initially such an increase is seen to be at a slower rate, but nearer to the critical load it increases rapidly before failure occurs [Fig.5 (*b*)]. Usually the failure is triggered by yielding under the combined effect of axial compression and maximum moment. It is seen that the end moments modify the behaviour of an axially loaded column in a way, similar to the initial bow type of imperfections.



(a) At constant axial load, P *(b) At constant end moments, M_0*

Fig. 5 Elastic Behaviour of Beam-Columns

Conventional first order linear elastic analyses of frames do not reflect these additional bending moments in beam-columns due to P-δ and P-Δ effects, since the equilibrium equations in these analyses methods are derived for the un-deformed structure. In the linear elastic analysis of a frame, the axial force and bending moments in a beam-column increase linearly, with the increase in the load on frame, as shown by straight line OA in Fig.3 drawn for short column. However, if we look at the equilibrium of the beam-column in the deformed configuration [Fig.4], (through a non-linear analysis), bending moments are magnified by $P\text{-}\Delta$ and $P\text{-}\delta$ effects in a nonlinear fashion, as the load increases.

The non-linear variation of the maximum bending moment, M_{max}, due to increase in the axial load, P, acting at a constant end eccentricity (M_0/P = constant end eccentricity) of a long column is also shown in Fig.3 (curve OB). When this non-linear P versus M loading curve intersects the failure envelope at B, the long beam-column would fail. It is seen that if the moment magnification of the long column due to P-δ and P-Δ effects were not considered, the compressive strength of the member would be obtained as P_0, whereas the actual compressive strength of the long beam-column is reduced to P_{cl}, due to the moment magnification effect. The corresponding linear analysis moment is M_o, whereas the actual magnified moment is M_{max}. The additional deflection and bending moment are due to the axial load acting on the deformed column as given below.

- in a column within a floor (P-δ effect) [Figs. 4(*a*) and 4 (*b*)]
- between the ends of the columns (sway) at adjacent floors (P-Δ effect) [Fig. 4(*c*)].

The magnified deflection and bending moment can be approximately obtained from the following equations.

$$\delta = \frac{\delta_0}{\left(1 - P/P_E\right)} \qquad (4a)$$

$$\Delta = \frac{\Delta_0}{\left(1 - P/P_E\right)} \qquad (4b)$$

$$M_{max} = \frac{C_m M_0}{\left(1 - P/P_E\right)} \qquad (4c)$$

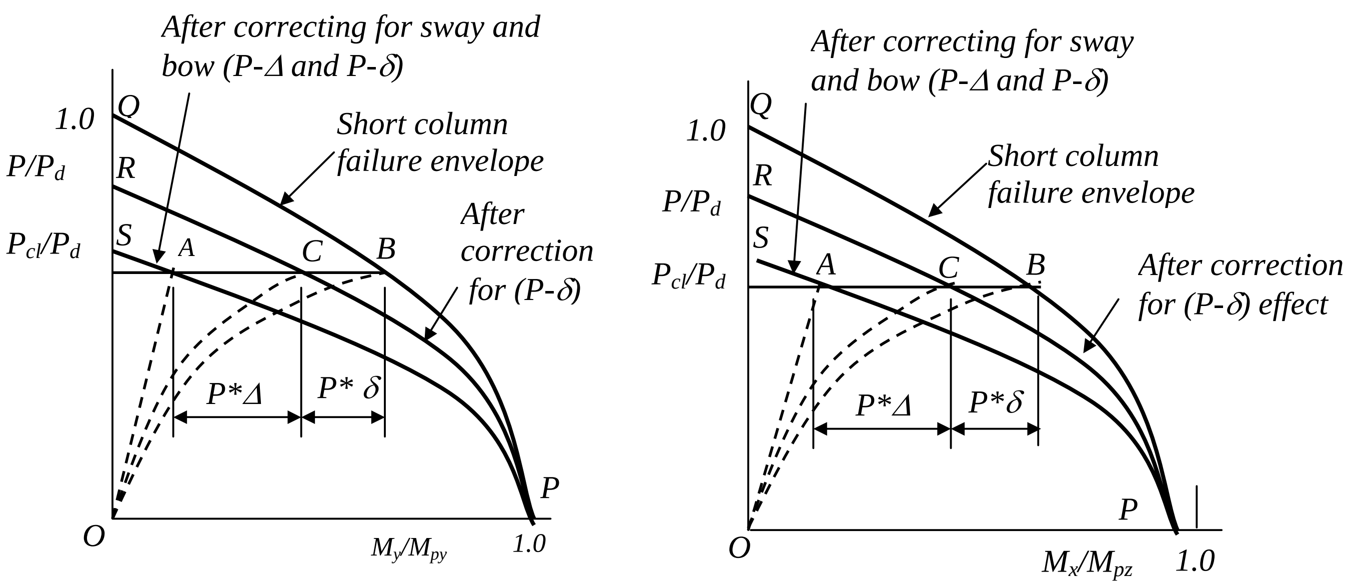
where C_m is a coefficient that accounts for the moment gradient effects explained in greater detail later in this chapter. P_E is the Euler buckling strength of the column in the plane of bending. It is seen that as the applied axial load approaches the Euler buckling load, both deflection, δ, and magnified moment, M_{max}, increase rapidly and tend to approach infinity, indicating that even if M_0 is very small, as P approaches P_E, failure is imminent.

3.1 Beam-columns at Ultimate Load

An axially loaded I section long column fails by buckling about the slender axis. The member (beam) bent about the major axis fail by either formation of plastic hinge at plastic moment M_{pz} or by lateral buckling at a value of bending moment less than M_{pz} depending upon the laterally unsupported length. The member bent about the minor axis fails by formation of plastic hinge at plastic moment, M_{py}.

A beam-column becomes axially loaded compression member when the eccentricity of the applied compression is equal to zero. When the eccentricity of the applied compression is very large (tending to infinity) the beam column tends to behave like a beam, since the axial compression effect is negligible. Thus these two cases define the two limits of a beam-column. In between, a beam-column covers a range of combination of axial load and bending moment. Due to this, various combinations of buckling and plastic failures are exhibited by beam-columns, depending upon the relative values of the axial force, bending moment, buckling strength and bending strength of the member. Further, the bending may be about the minor axis only, causing flexural yielding type of failure or about the major axis only, causing torsional flexural buckling, or a combination of bending moments about both the axes.

Figs. 6(*a*) and 6(*b*) show the strength of typical beam-columns made of I sections, subjected to axial compression and uniaxial bending about the minor and major axis, respectively. The solid curves represent the strength envelope of beam-columns in a frame, considering the different long column and sway effects. The curves PBQ in Figs. 6(*a*) and 6(*b*) represent the strength envelopes of a stub column without considering the $P\text{-}\delta$ and $P\text{-}\Delta$ effects. Therefore, if these short column strength envelopes are used, the actual bending moments and axial force used in evaluating the strength should be based on a non-linear analysis accounting for the P-δ and P-Δ effects. Thus the loading path from such an analysis would be represented by the dashed curve OB. Similarly, the strength envelopes PCR account for the P-δ effects only. Hence, if these strength envelope curves are used, actual moment and axial force evaluation should be based on an analysis method that accounts for P-Δ effects. The loading path (P versus M) from such an analysis is represented by dashed line OC. The strength envelopes PAS account for both $P\text{-}\delta$ and $P\text{-}\Delta$ effects. Hence a linear analysis is adequate to obtain the axial forces and moments, while using these strength curves.



(a) Minor axis bending

(b) Major axis bending

Fig. 6 Uniaxial Bending of Slender Beam-Columns

It is seen in Figs. 6(*a*) and 6(*b*) that the strength under axial compression decreases when the $P\text{-}\delta$ and $P\text{-}\Delta$ effects are considered. It is also seen that under pure bending, the major axis bending strength is affected due to the lateral buckling and that pure bending strength can be less than the plastic moment capacity M_{pz}. The bending strength, when the axial compression is zero, is given by M_z, which is less than M_{pz}. The dashed curves represent the loading path corresponding to different levels of analysis. The dashed curves OA represent the linear analysis path and hence their intersections with the strength envelopes corrected for P-δ and P-Δ effects (PAS), give the member strengths, P_{cl}. The dashed curves OB represent the nonlinear analysis paths, considering P-δ and P-Δ effects and hence intersections of these curves with the short column strength curve (PBQ) give the member strengths, P_{cl}. The dashed curves OC represent the loading paths from nonlinear analysis considering P-Δ effects only and hence their intersections with the strength curves (PCR) corrected for P-δ effect give the member strengths, P_{cl}.

3.2 Effects of Slenderness Ratio and Axial Force on Modes of Failure

Beam-columns may fail by flexural yielding or torsional flexural buckling. The actual mode of failure would depend upon the magnitude of the axial load and eccentricity as well as the slenderness ratio. The sub-ultimate and failure behaviour of beam-columns, as affected by different parameters, are briefly reviewed in the following sections (Dowling et al., 1988).

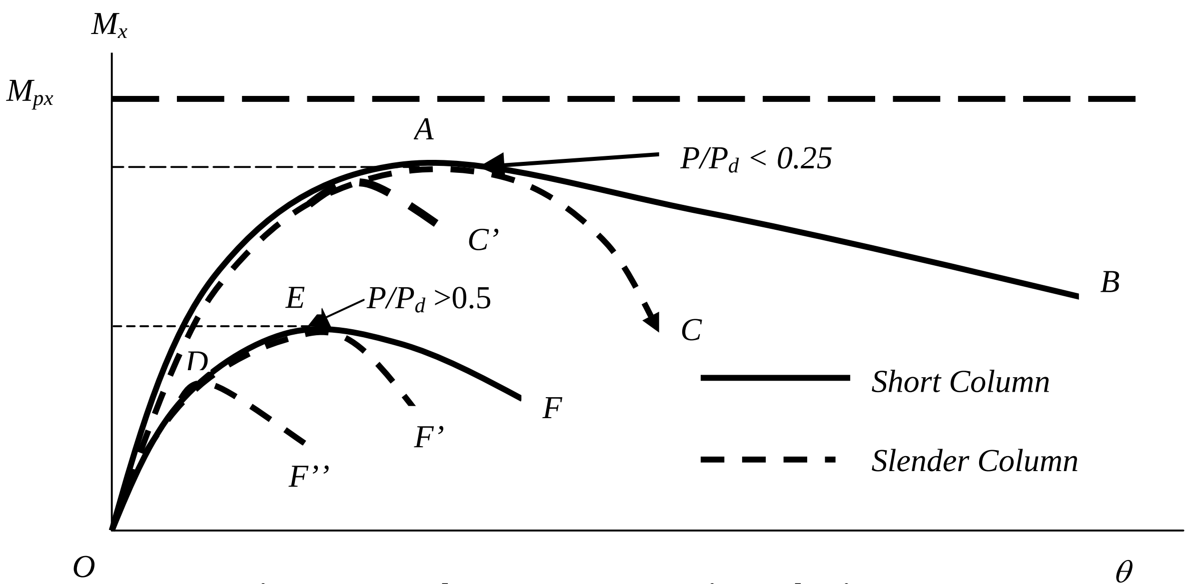


Fig. 7 Beam-column Moment Rotation Behaviour

3.2.1 Low axial load ratio (P/P_d< 0.33)

<u>Beam-columns having lower slenderness ratios (λ/r <50)</u>: When subjected to moments about the major axis at both ends of the beam, the moment-curvature relationship at the sub-ultimate stage may be linear or non-linear depending upon whether the axial force is applied first followed by bending moment or both of them are increased proportionately (Figs. 5). The deformation is only in the plane of bending moment. At the penultimate stage, yielding of the compression flange occurs first, which spreads through the section

on further loading. The ultimate strength is reached, when the plastic hinge is formed at one or both the ends. Thus the failure is due to the section strength being reached at one or both the ends. Under proportional loading, the loading path is indicated by the line OAB in Fig. 7. It is seen that there would be a small reduction in the moment capacity below M_{pz} due to the presence of the axial compression in the member. Further, there would be a reduction in the moment (unloading) beyond point A, with increasing end rotation, due to the spread of plasticity from the end sections to other sections along the length.

Beam-columns having a higher slenderness ratio (λ/r.>80): A more slender column, under smaller axial compression combined with end moments as before, would fail by buckling out-of-the-plane of the bending moment. If bending is predominant, lateral buckling of compression flange as in unrestrained beams occurs (OAC in Fig. 7). The axial force could cause minor axis deformation and hence the failure can be by minor axis bending and twisting (OC'), at moments below the full in plane strength obtained in the case of short/stocky columns. The moment rotation behaviour at the ultimate stage is indicated by dashed line in Fig.7. The failure would be after plastic hinge formation unless the slenderness ratio is very large.

3.2.2 High axial load ratio (P/P_d > 0.5)

Beam-columns having a lower slenderness ratio (λ/r <50): Under high axial load combined with bending moment, yielding can occur over a larger segment of the member, due to combined axial stress, bending stress and residual stress. Moment magnification at the mid-length of the column occurs due to single curvature bending deflection caused by equal end moments [Fig.4 (*a*)]. In the case of short/stocky member, the failure is due to the yield strength being reached at a section over the length of the member under combined axial force and magnified bending moment. The corresponding curve is shown by line ODEF in Fig.7. The main differences in the behaviour of stocky beam-column under larger axial compression, compared to smaller axial compression are the moment magnification, larger reduction in moment capacity due to larger axial compression and the drastic unloading in the penultimate stage (EF in Fig. 7).

Beam-columns having a higher slenderness ratio (λ/r.>80): In the case of slender beam-columns with a larger axial compression, the P-δ effect is larger both in the plane of and out of plane of the moment. In longer beam-columns, the moment may drop drastically when yielding starts under combined axial compression and magnified moment (OEF'). The weak axis buckling/flexural torsional buckling, causing out-of-plane deformation, could occur earlier than that corresponding to the section strength under combined axial force and magnified moment (ODF" in Fig. 7).

Thus design of beam-columns having higher slenderness ratio requires investigation of in- plane bending failure by flexural yielding and out-of-plane buckling failure. The behavior of beam-columns subject to bending about the minor axis is similar to that subjected to major axis bending as discussed, but for the following differences:

- In the case of slender members under smaller axial load, there is very little reduction of moment capacity below M_p, since the lateral torsional buckling is not a problem in weak axis bending.
- The moment magnification is larger in the case of beam-columns bending about the weak axis.
- The failure of short/stocky members is either due to section strength being reached at the ends (under smaller axial load) or at the section of larger magnified moment (under larger axial load).
- The failure of even slender members is due to buckling about the weak axis only and no torsional deformation is experienced.
- The *M-P* failure envelope varies as shown in Fig.6 (*a*), due to the variations of slenderness ratio and axial force.

3.3 Beam-Column under Biaxial Bending

The ultimate behaviour of beam-columns under biaxial bending is complicated by the effect of plastification, moment magnification and lateral torsional buckling. Typical failure envelope diagram is as shown in Fig.8. It is seen that the increase in the slenderness ratio of the member tends to reduce the strength of the member, except in the case of nearly pure bending about the weak axis (y-axis). Further, but for very small axial force ranges, increases in the axial compression tend to decrease the bending strength about both axes.

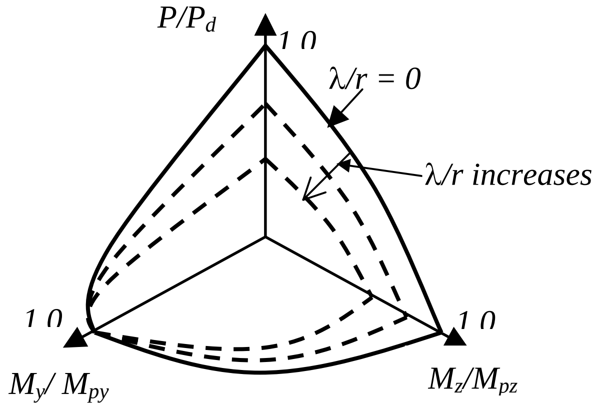


Fig. 8 beam-columns under Biaxial Bending

4.0 SUMMARY

Behaviour of beam-columns, which are members subjected to axial compression and bending simultaneously, was discussed in this chapter. The following were the main issues discussed in the chapter.

- The moment magnification due to $P\text{-}\delta$ and $P\text{-}\Delta$ effects are important considerations affecting their strength, particularly in case of slender members subjected to larger axial compression.
- Under major axis bending lateral buckling is also an important consideration.

- Flexural yielding, flexural buckling, torsional flexural buckling are the different modes of failure, depending upon the slenderness ratio, axis of bending and extent of axial compression.

In the next chapter, II of this topic, evaluation of strength of beam-columns will be dealt with and steps in designing beam-columns will be presented supported by an example.

5.0 REFERENCES

1. Dowling P.J, Knowles, P.R. and Owens, G.W., “Structural Steel Design”, Butterworths, London, 1998.