14 DESIGN OF BEAM-COLUMNS - II

1.0 INTRODUCTION

Beam-columns are members subjected to combined bending and axial compression. Their behaviour under uniaxial bending, biaxial bending and torsional flexural buckling were discussed in Part I on this topic in the previous chapter. It was shown that a range of behaviour varying from flexural yielding to torsional flexural or flexural buckling is possible.

In this chapter evaluation of strength of beam-columns is discussed. The steps in the analysis of strength of beam-column are presented along with an example.

2.0 STRENGTH OF BEAM-COLUMNS

The discussions in the part I of this topic, clearly indicated that the behaviour of beam-columns is fairly complex, particularly at the ultimate stage and hence exact evaluation of the strength would require fairly complex analysis. However, for design purposes, simplified equations are available, using which it is possible to obtain the strength of members, conservatively. These are discussed below.

2.1 Modes of Failure

The following are the possible modes of failure of beam-columns

2.1.1 *Local section failure*

This is usually encountered in the case of short, stocky beam columns ($\lambda/r << 50$) with relatively smaller axial compression ratio ($P/P_d < 0.33$) and beam-columns bent in reverse curvature.

- The strength of the end section reached under combined axial force and bending moment, governs the failure.
- The strength of the section may be governed by plastic buckling of plate elements in the case of plastic, compact and semi-compact sections or the elastic buckling of plate elements in the case of slender sections (see the chapter on plate buckling).

2.1.2 Overall instability failure under flexural yielding

This type of failure is encountered in the case of all members subjected to larger compression ($P/P_d > 0.5$) and single curvature bending about the minor axis as well as not very slender members subjected to axial compression and single curvature bending about the major axis.

- The member fails by reaching the strength of the member at a section over the length of the member, under the combined axial compression and magnified bending moment.
- In the case of weak axis bending of slender members ($\lambda/r > 80$), the failure may be by weak axis buckling, or failure of the maximum moment section under the combined effect of axial force and magnified moment.
- The section failure may be due to elastic or plastic plate buckling depending on the slenderness ratio (b/t) of the plate (See the chapter on plate buckling).

2.1.3 Overall instability by torsional flexural buckling

This is common in slender members ($\lambda/r>80$) subjected to large compression ($P/P_d>0.5$) and uniaxial bending about the major axis or biaxial bending.

- At the ultimate stage the member undergoes biaxial bending and torsional instability mode of failure.

2.1.4 Design Equations as per IS: 800

The design code specifies, as given below, the linear interaction equations to check the section strength to prevent local section failure as well as member failure by flexural yielding and torsional flexural buckling. These are conservative simplifications of the non-linear failure envelopes discussed in the previous chapter.

2.2.1 *Local section failure*

The Indian Standard Specification IS: 800 clearly deals with all types of members and mainly classifies the members in two groups, namely Plastic and *Compact Sections* and *Semi-Compact* sections. The strength of *Slender Members* may be analysed by following the procedure discussed in the chapters on *cold-formed steel members.*

IS: 800 states that under combined axial force and bending moment section strength as governed by material failure and member strength as governed by buckling failure have to be checked as given below;

1. *Section Strength*

<u>*Plastic and Compact Sections*</u> ─ In the design of members subjected to combined axial force (tension or compression) and bending moment, the following should be satisfied

$$\left(\frac{M_y}{M_{ndy}}\right)^{\alpha_1} + \left(\frac{M_z}{M_{ndz}}\right)^{\alpha_2} \leq 1.0 \qquad (1)$$

Conservatively, the following equation may be used under combined axial force and bending moment

$$\frac{P}{P_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \leq 1.0 \qquad (2)$$

where

M_y, M_z = factored applied moments about the minor and major axis of the cross section, respectively

M_{ndy}, M_{ndz} = design reduced flexural strength under combined axial force and the respective uniaxial moment acting alone

P= factored applied axial force

P_d = design strength in compression due to yielding given by

$$P_d = A_g f_y / \gamma_{m0}$$

M_{dy}, M_{dz} = design strength under corresponding moment acting alone

A_g = gross area of the cross section

α_1, α_2 = constants as given in Table 1

$n = N / N_d$

Table: 1 Constants α_1 and α_2

Section	α_1	α_2
I and Channel	$5n \geq 1$	2
Circular tubes	2	2
Rectangular tubes	$1.66/(1-1.13n^2) \leq 6$	$1.66/(1-1.13n^2) \leq 6$
Solid rectangles	$1.73+1.8n^3$	$1.73+1.8\,n^3$

For plastic and compact sections without bolts holes, the following approximations may be used for evaluating M_{ndy} and M_{ndz}:

a) *Plates*

$$M_{nd} = M_d (1-n^2)$$

b) *Welded I or H sections*

$$M_{ndy} = M_{dy}\left[1-\left(\frac{n-a}{1-a}\right)^2\right] \leq M_{dy}$$

$$M_{ndz} = M_{dz}\,(1-n) / (1-0.5a) \leq M_{dz}$$

where $n = P / P_d$ and $a = (A - 2\,b\,t_f) / A \leq 0.5$

c) *For standard I or H sections*

for $n \leq 0.2$ $\quad M_{ndy} = M_{dy}$

for $n>0.2$ $\quad M_{ndy} = 1.56\,M_{dy}\,(1-n)\,(n+0.6)$

$\quad M_{ndz} = 1.11\,M_{dz}\,(1-n) \leq M_{dz}$

d) *For Rectangular Hollow sections and Welded Box sections* –

When the section is symmetric about both axes and without bolt holes

$$M_{ndy} = M_{dy}\,(1-n) / (1-0.5a_f) \leq M_{dy}$$

$M_{ndz} = M_{dz}\,(1-n)\,/\,(1-0.5a_w) \leq M_{dz}$

where

$a_w = (A - 2\,b\,t_f)\,/\,A \quad \leq 0.5$

$a_f = (A - 2\,h\,t_w)\,/\,A \quad \leq 0.5$

e) *Circular Hollow Tubes without Bolt Holes*

$M_{nd} = 1.04\,M_d\,(1-n^{1.7}) \leq M_d$

<u>*Semi-compact section*</u> – In the absence of high shear force semi-compact section design is satisfactory under combined axial force and bending, if the maximum longitudinal stress under combined axial force and bending, *fBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB*$_{x,}$ satisfies the following criteria.

$f_x \leq f_y / \gamma_{m0}$

For cross section without holes, the above criteria reduces to

$$\frac{P}{P_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \leq 1.0 \qquad (3)$$

where

P_d, M_{dy}, M_{dz} are as defined earlier

2. *Overall Member Strength*

Members subjected to combined axial compression and moment shall be checked for overall buckling failure as given below:

$$\frac{P}{P_{dy}} + k_y \frac{C_{my} M_y}{M_{dy}} + k_{LT} \frac{M_z}{M_{dz}} \leq 1.0$$

$$\frac{P}{P_{dz}} + 0.6 k_y \frac{C_{my} M_y}{M_{dy}} + k_z \frac{C_{mz} M_z}{M_{dz}} \leq 1.0 \qquad (4)$$

where

C_{my}, C_{mz} =	Equivalent uniform moment factor as per Table 2
P =	applied axial tension or compression under factored load
M_y, M_z =	maximum factored applied bending moments about *y* and *z*-axis of the member, respectively.
P_{dy}, P_{dz} =	design strength under axial tension or compression as governed by buckling about minor (y) and major (z) axis respectively.
M_{dy}, M_{dz} =	design bending strength about y (minor) or z (major) axis of the cross section

$K_y = \quad 1+(\lambda_y-0.2)n_y \leq 1+0.8\,n_y$

$K_z = \quad 1+(\lambda_z-0.2)n_z \leq 1+0.8\,n_z$

$$K_{LT} = 1 - \frac{0.1\lambda_{LT} n_y}{\left(C_{mLT} - 0.25\right)} \geq 1 - \frac{0.1 n_y}{\left(C_{mLT} - 0.25\right)}$$

where,

n_y, n_z = ratio of actual applied axial force to the design axial strength for buckling about the y and z axis, respectively and

C_{mLT} = Equivalent uniform moment factor for lateral torsional buckling as per Table 2 corresponding to the actual moment gradient between lateral supports against torsional deformation in the critical region under consideration.

More accurate evaluation of beam-column strength is possible by resorting to non-linear P-Δ analysis. In this case, the actual axial compression and bending moments as obtained from such an analysis may be used in the interaction equation and the sway effects may be disregarded in evaluation of P_u, P_{ex} and P_{ey}. These methods of analysis and design are beyond the scope of this chapter and are not discussed herein.

3.0 STEPS IN ANALYSING A BEAM-COLUMN

(i) Calculate the cross section properties.

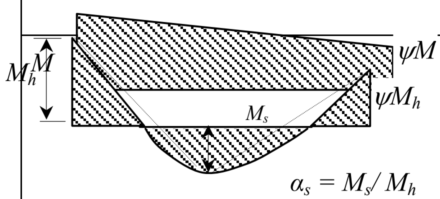
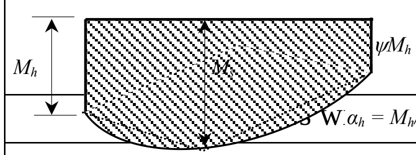
Area, principal axes moments of inertia, section moduli, radii of gyration, effective lengths and slenderness ratios.

(ii) Evaluate the type of section based on the (*b/t*) ratio of the plate elements, as plastic, compact, semi-compact, or slender.

(iii) Check for resistance of the cross-section under the combined effects as governed by yielding (Eq. 1,2 or 3).

(iv) Check for resistance of member under the combined effects as governed by buckling (Eq. 4).

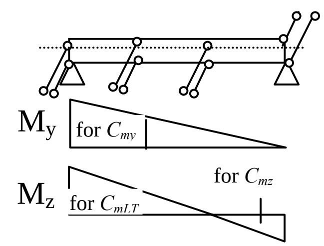
Table: 2 Equivalent Uniform Moment Factor

Bending moment diagram	**Range**		C_{my}, C_{mz}, C_{mLT} *Uniform loading*	C_{my}, C_{mz}, C_{mLT} *Concentrated load*
	$-1 \le \psi \le 1$		$0.6 + 0.4\,\psi \ge 0.4$	
	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0.2 + 0.8\,\alpha_s \ge 0.4$	$0.2 + 0.8\,\alpha_s \ge 0.4$
	$-1 \le \alpha_s \le 0$	$0 \le \psi \le 1$	$0.1 - 0.8\,\alpha_s \ge 0.4$	$-0.8\,\alpha_s \ge 0.4$
		$-1 \le \psi \le 0$	$0.1(1-\psi) - 0.8\,\alpha_s \ge 0.4$	$0.2(1-\psi) - 0.8\,\alpha_s \ge 0.4$
	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0.095 - 0.05\,\alpha_h$	$0.90 + 0.10\,\alpha_h$
	$-1 \le \alpha_s \le 0$	$0 \le \psi \le 1$	$0.095 + 0.05\,\alpha_h$	$0.90 + 0.10\,\alpha_h$
		$-1 \le \psi \le 0$	$0.95 + 0.05\,\alpha_h\,(1+2\,\psi)$	$0.90 + 0.05\,\alpha_h\,(1+2\,\psi)$

…s w… buckling mode the equivalent uniform moment factor $C_{my} = C_{mz} = 0.9$.

C_{my}, C_{mz}, C_{mLT} shall be obtained according to the bending moment diagram between the relevant braced points

Moment factor	*Bending axis*	*Points braced in direction*
C_{my}	*z-z*	*y-y*
C_{mz}	*y-y*	*z-z*
C_{mLT}	*z-z*	*z-z*



4.0 SUMMARY

This chapter presented equations for the design of beam-columns and an example design. The behaviour and design of beam-columns are contained in the two parts on this topic. The following are the important points discussed in these chapters.

- The beam-column may fail by reaching either the ultimate strength of the section (in the case of smaller axial load and shorter members) or by the buckling strength as governed by weak axis buckling or lateral torsional buckling.
- At lower loads, the failure is likely to be after the formation of the plastic hinges, especially in the case of shorter members.
- In slender beam-columns with larger axial compression, either weak axis or lateral torsional buckling would control failure.
- The interaction formulae given for the design are conservative and simple, considering the complicated nature of beam-column failure.
- In the design of beam-columns in frames, the magnification of moment due to P-δ and P-Δ effects are to be considered.

5.0 REFERENCES

1. IS: 800(2007), “General Construction in Steel – Code of Practice”, Bureau of Indian Standards, New Delhi, 2007.

2. Dowling P.J, Knowles and Owens, G.W., “Structural Steel Design”, Butterworth, London, 1998.

3. Eurocode 3: 1992, “Design of Steel structures”, British Standards Institution.