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*Analyse the building frame shown in Fig. A using portal method.*

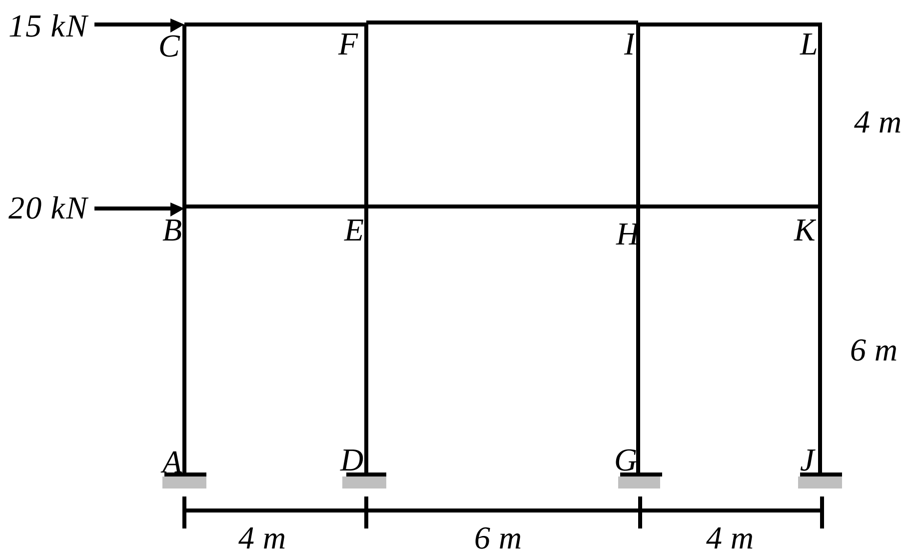


***Fig. A***

***Top storey:***

***(i) Column Shears:***

*Shear in columns of the top storey is obtained by considering the free body diagram shown in Fig. A .1*

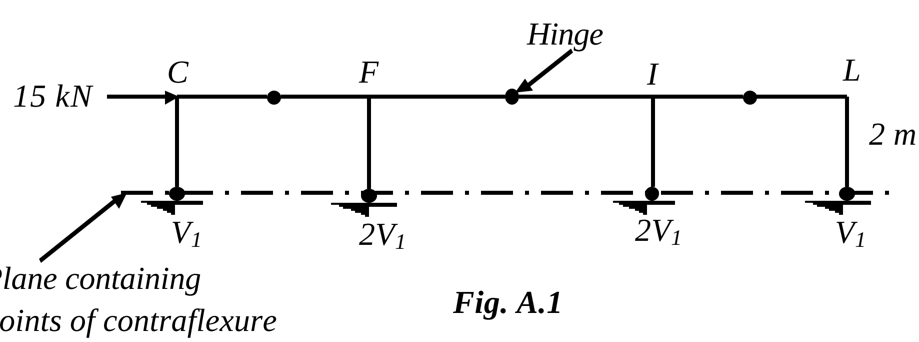


***Fig. A.1***

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$V_1 + 2V_1 + 2V_1 + V_1 = 15$ kN [Assumption 3]

*Shear in end column,* $V_1 = 2.5$ kN

*Shear in middle columns,* $2V_1 = 5.0$ kN

*Thus, shear in columns are:*

| ***Column*** | ***Shear (kN)*** |
|---|---|
| *CB* | *2.5* |
| *FE* | *5.0* |
| *IH* | *5.0* |
| *LK* | *2.5* |

***(ii) Column moments:***

*Column moments are found by multiplying column shear and half the height of column as shown below:*

| ***Column*** | ***Shear (kN)*** | ***Moment (kN-m)*** |
|---|---|---|
| *CB* | *2.5* | *2.5 * 2 = 5.0* |
| *FE* | *5.0* | *5.0 * 2 = 10.0* |
| *IH* | *5.0* | *5.0 * 2 = 10.0* |
| *LK* | *2.5* | *2.5 * 2 = 5.0* |

***(iii) Girder Moments:***

*At any joint, sum of the girder moments is equal to the sum of the column moments. Starting from left corner of the frame, C*

*Joint C:*

$M_{CB} = M_{CF} = 5$ kN-m

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*Joint F:*

$M_{FC} + M_{FI} = M_{FE}$

$M_{FI} = M_{FE} - M_{FC} = 10 - 5 = 5$ *kN-m*

*Joint I:*

$M_{IF} + M_{IL} = M_{IH}$
$M_{IL} = M_{IH} - M_{IF} = 10\text{-}5 = 5.0$ *kN-m*

***(iv) Girder shears:***

*Girder shear = Girder Moment / (span/2)*

| *Span* | *Span/2 (m)* | *Moment (kN-m)* | *Shear (kN)* |
|---|---|---|---|
| *CF* | *2.0* | *5.0* | *2.50* |
| *FI* | *3.0* | *5.0* | *1.67* |
| *IL* | *2.0* | *5.0* | *2.50* |

*(v)* ***Column axial forces:*** *(See Fig. A .2)*

*Axial force on a column is determined by summing up the girder shears and other axial forces at each joint.*

*Starting from the left corner of the frame, we have*

*Joint C:* $SF_y = 0$
$F_{CB} = V_{CF} = 2.5$ *kN*

*Joint F:* $SF_y = 0$
$F_{EF} + V_{FC} = V_{FI}$
$F_{EF} = V_{FI} - V_{FC} = 1.67\text{-}2.5 = -0.83$ *kN*

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*Joint I :* $SF_y = 0$

$V_{IF} = F_{IH} + V_{IL}$

$F_{IH} = V_{IF} - V_{IL} = 1.67 - 2.5 = -0.83\ kN$

*Joint L :* $SF_y = 0$

$F_{KL} = V_{LI} = 2.5\ kN$

***Ground storey:***

*(i)* ***Column shears:***

***Shear in the columns of ground storey is calculated in similar way to the top storey. The plane containing points of contraflexure will pass through the half the height of the ground storey. Then shear in columns is calculated as***

$SF_H = 0$

$6V_1 = 35\ kN.$

$V_1 = 35/6 = 5.8\ kN.$

*Column Shears;*

$V_{AB} = 5.8\ kN$ ; $V_{DE} = 11.7\ kN$

$V_{JK} = 5.8\ kN$ ; $V_{GH} = 11.7\ kN.$

***(ii)*** ***Column moments:***

| *Column* | *Length/2 (m)* | *Shear (kN)* | *Moment (kN-m)* |
|---|---|---|---|
| *AB* | *3.0* | *5.8* | *17.5* |
| *DE* | *3.0* | *11.7* | *35.0* |
| *GH* | *3.0* | *11.7* | *35.0* |
| *JK* | *3.0* | *5.8* | *17.5* |

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***(iii) Girder moments:***

*Joint B:*

$M_{BE} = M_{BC} + M_{BA} = 5.0 + 17.5 = 22.5$ kN-m

*Joint E:*

$M_{EH} = M_{EF} + M_{ED} - M_{EB}$

$= 10 + 35.0 - 22.5 = 22.5$ kN-m

*Joint H:*

$M_{HK} = M_{IH} + M_{HG} - M_{HE}$

$= 10 + 35.0 - 22.5 = 22.5$ kN-m

*Joint K:*

$M_{KH} = M_{KL} + M_{JK} = 5 + 17.5 = 22.5$ kN-m

***(iv) Girder shears:***

| Beam | Span/2 (m) | Moment (kN-m) | Shear (kN) |
|---|---|---|---|
| BE | 2.0 | 22.5 | 11.3 |
| EH | 3.0 | 22.5 | 7.5 |
| HK | 2.0 | 22.5 | 11.3 |

***(v) Column axial forces:*** *(Consider the Fig. A .2)*

*Joint B:*

$\Sigma F_V = 0$

$F_{AB} = F_{BC} + V_{BE} = 2.5 + 11.3 = 13.8$ kN

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*Joint E:* $F_{ED} = F_{EF} + V_{EH} - V_{EB} = -0.83 + 7.5 - 11.3$

$= -4.6$ kN

*Joint H:* $F_{GH} = V_{EH} + F_{IH} - V_{HK} = 7.5 - 0.83 - 11.3$

$= -4.6$ kN

*Joint K:* $F_{JK} = V_{KH} + F_{LK} = 11.3 + 2.5$

$= 13.8$ kN

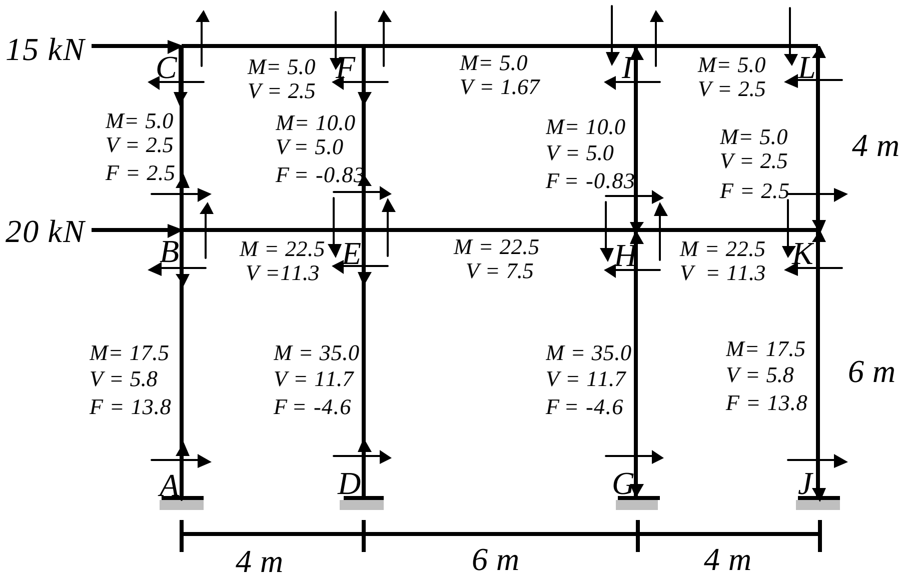


***Fig. A. 2. Axial forces in columns and shear forces in members***

*M – Moment in kN-m*
*V – Shear in kN*
*F – Axial force in kN*

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*Problem 2:*
*Analyse the building frame shown in previous example (Fig. A) using Cantilever method. Assume cross-sectional areas of all the columns as equal.*

***Top storey:***

***(1) Location of centroidal line of columns of the storey:***

*Let the area of each column be A and x be distance to the centre of gavity of columns shown in Fig. B.1*

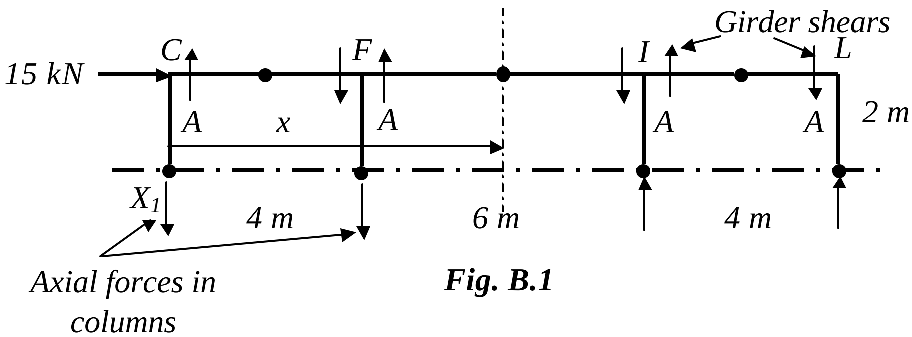


***Fig. B.1***

*Take moments about column BC*

$$x = \frac{0\times A + 4\times A + 10\times A + 14\times A}{4A} = \frac{28A}{4A} = 7m$$

(2) ***Column axial forces:*** *(See Fig. B.1)*

*In cantilever method, it is assumed that the axial forces in the columns are proportional to the horizontal distance from the center of gravity of the columns in the storey.*

*Say,* $F_{BC} = F$

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$$\frac{F_{BC}}{F_{EF}} = \frac{7}{3} \quad \Rightarrow \quad F_{EF} = \frac{3}{7}F_{BC} = 3/7\,F$$

$$F_{HI} = \frac{3}{7}F \quad ; \qquad F_{KL} = F$$

*Take moments about $X_1$,*

$$15 \times 2 + F_{EF} \times 4 - 10 \times F_{HI} - 14\,F_{KL} = 0$$

$$15 \times 2 + \frac{3}{7}F \times 4 - 10 \times \frac{3}{7}F - 14\,F = 0$$

$$30 - \frac{18}{7}F - 14F = 0$$

$$30 = \frac{116}{7}F$$

$$F = \frac{210}{116} = 1.81\,kN$$

$$F_{BC} = 1.81\,kN \qquad F_{EF} = 0.78\,kN$$

$$F_{HI} = 0.78\,kN \qquad F_{KL} = 1.81\,kN$$

*(3)* ***Shear forces at the ends of beams:*** *(See Fig. B.1)*

*Joint C:* $\sum F_y = 0 \Rightarrow V_{CF} = F_{BC} = 1.81\,kN$

*Joint F:* $V_{FC} + F_{EF} = V_{FI}$

$$V_{FI} = 1.81 + 0.78 = 2.59\ kN \qquad (\because\ V_{FC} = V_{CF})$$

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*Joint I:* $V_{IL} = V_{IF} - F_{HI} = 2.59 - 0.78 = 1.81$ kN ($V_{IF} = V_{FI}$)

*Joint L:* $V_{LI} = F_{KL} = 1.81$ kN

***(4) Girder moments:***

*Girder moment = Girder shear * Span/2*

| Girder | Shear (kN) | Span /2 (m) | Moment (kN-m) |
|---|---|---|---|
| CF | 1.81 | 2.0 | 3.62 |
| FI | 2.59 | 3.0 | 7.77 |
| IL | 1.81 | 2.0 | 3.62 |

*(5)* ***Column moments:*** *At each joint, sum of girder moments equals to sum of column moments. Consider joints from left corner of the floor.*

*Joint C:* $M_{CB} = M_{CF} = 3.62$ kN-m

*Joint F:* $M_{FE} = M_{FC} + M_{FI}$
$= 3.62 + 7.77 = 16.3$ kN-m

*Joint I:* $M_{IH} = M_{IF} + M_{IL}$
$= 7.77 + 3.62 = 16.3$ kN-m

*Joint L:* $M_{LI} = M_{LK} = 3.62$ kN-m

***(6) Column Shears:***

*Column Shear = Column moment / (Length/2)*

| Column | Moment (kN-m) | Length/2 (m) | Shear (kN) |
|---|---|---|---|
| BC | 3.62 | 2.0 | 1.81 |
| EF | 11.4 | 2.0 | 5.70 |
| HI | 11.4 | 2.0 | 5.70 |
| KL | 3.62 | 2.0 | 1.81 |

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***Ground storey:***

*(i) Location of centroidal line of columns of the storey:*

*Consider the following free body diagram shown Fig. B.2*

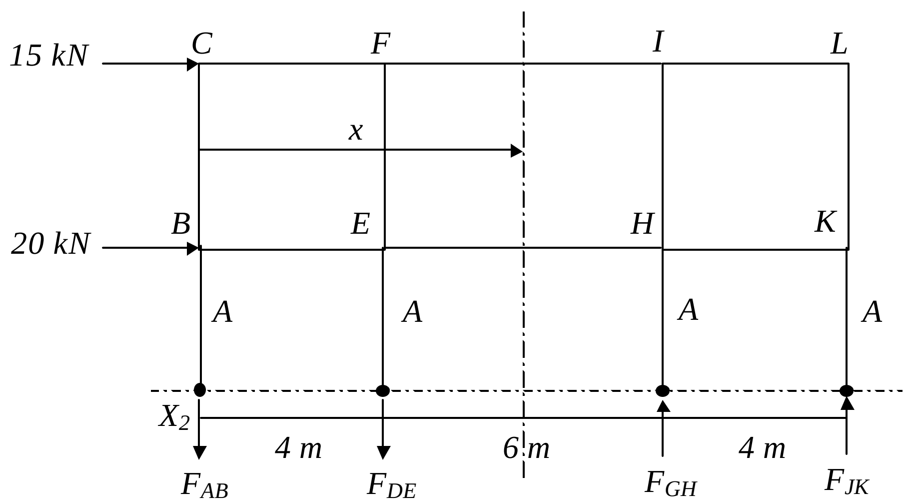


***Fig. B. 2***

*Let the area of each column be 'A' and x be the distance to the centre of gravity of columns as shown in Fig. B.2*

$$x = \frac{0 \times A + 4 \times A + 10 \times A + 14 \times A}{4A} = 7\ m$$

***(ii) Column axial forces:***

*Say,* $F_{AB} = F$

*Then,* $F_{DE} = 3/7\ F$ ; $F_{GA} = 3/7\ F$ *and* $F_{JK} = F$

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Taking moments about $X_2$,

$$15\times(4+6/2)+20\times(6/2)+F_{DE}\times4-F_{GH}\times10-14\times F_{JK}=0$$

$$\Rightarrow 105+60+\frac{12}{7}F-\frac{30}{7}F-14F=0$$

$$\Rightarrow F=\frac{165\times7}{116}=10.0kN$$

*Axial forces in columns are,*

$F_{AB}$ = 10.0 kN ; $F_{DE}$ = 4.3 kN
$F_{GH}$ = 4.3 kN ; $F_{JK}$ = 10.0 kN.

*(iii)* ***Beam shears:*** *[See Fig B.3]*

*Joint B:* $F_{AB} = V_{BE} + F_{BC}$
$V_{BE} = 10.0 – 1.81 = 8.2$

*Joint E:* $V_{EB} + F_{DE} = V_{EH} + F_{EF}$
$V_{EH} = 8.2+4.3-0.78$
$= 11.7$ kN ($V_{EB} = V_{BE}$)

*Joint H:* $V_{HK} = V_{HE} - F_{GH} + F_{HI}$
$V_{HK} = 11.7-4.3+0.78$
$= 8.2$ kN ($V_{HE} = V_{EH}$)

*Joint K:* $V_{KH} = F_{JK} - F_{KL} = 10.0 – 1.81$

$= 8.2$ kN

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***(iv) Girder Moments:***

| *Span* | *Length/2 (m)* | *Shear (kN)* | *Moment (kN-m)* |
|---|---|---|---|
| *BE* | *2.0* | *8.2* | *16.4* |
| *EH* | *3.0* | *11.7* | *35.1* |
| *HK* | *2.0* | *8.2* | *16.4* |

***(v) Column moments:***

*At each joint sum of column moments equals to sum of girder moments*

*Joint B:*

$M_{BC} + M_{BA} = M_{BE}$

$M_{BA} = M_{BE} - M_{BC}$

$= 16.4-3.62=12.8$ kN-m

*Joint E:*

$M_{EB} + M_{EH} = M_{EF} + M_{ED}$

$M_{ED} = M_{EB} + M_{EH} - M_{EF}$

$= 16.4+35.1-11.4=40.1$ kN-m

*Joint H:*

$M_{HE} + M_{HK} = M_{HI} + M_{HG}$

$M_{HG} = M_{HE} + M_{HK} - M_{HI}$

$=35.1+16.4-11.4=40.1$ kN-m

*Joint K:*

$M_{KH} = M_{KL} + M_{JK}$

$M_{JK} = M_{KH} - M_{KL} = 16.4-3.62$

$= 12.8$ kN-m

***(vi) Column shears:***

| ***Span*** | ***Length/2 (m)*** | ***Moment (kN-m)*** | ***Shear (kN)*** |
|---|---|---|---|
| *AB* | *3.0* | *12.8* | *4.3* |
| *DE* | *3.0* | *40.1* | *13.3* |
| *GH* | *3.0* | *40.1* | *13.3* |
| *JK* | *3.0* | *12.8* | *4.3* |

| | | | |
|---|---|---|---|
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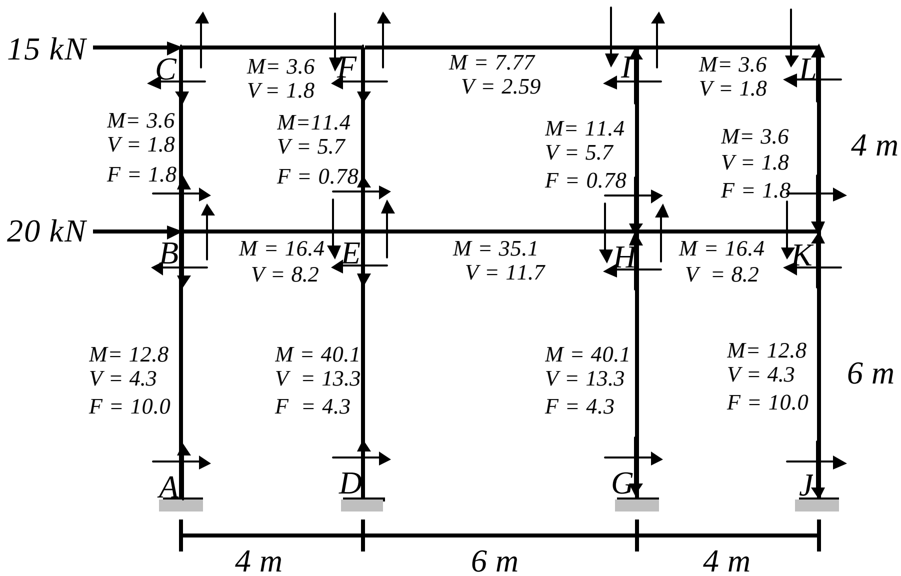


***Fig. B. 3. Axial forces in columns and shear forces in members***

*M – Moment in kN-m*
*V – Shear in kN*
*F – Axial force in kN*

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*Problem 3:*
*Determine the moments at the ends of columns and beams in the building frame shown in Fig. C by factor method. The relative stiffness factors (k) are mentioned in figure.*

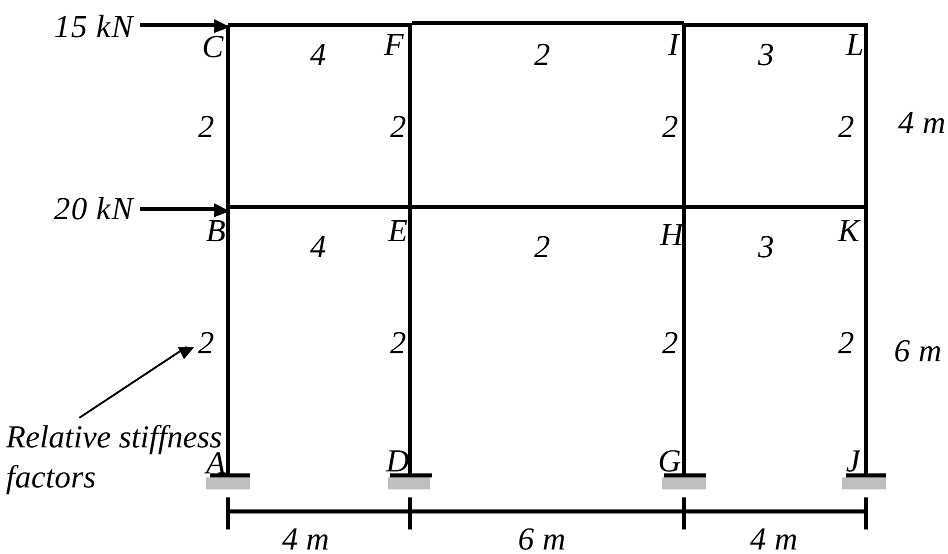


***Fig. C***

***(1) Girder factors:***

*Girder factor, g* = $\dfrac{\text{Sum of column relative stiffness factors at the joint}}{\text{Sum of total relative stiffness factors at that joint.}}$

*Joint C:*

$$g_C = \frac{2}{2+4} = 0.33$$

*Joint F:*

$$g_F = \frac{2}{4+2+2} = 0.25$$

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*Joint I:*

$$g_I = \frac{2}{2+2+3} = 0.286$$

*Joint L:*

$$g_L = \frac{2}{2+3} = 0.4$$

*Joint B:*

$$g_B = \frac{2+2}{2+2+4} = 0.5$$

*Joint E:*

$$g_E = \frac{2+2}{2+2+2+4} = 0.4$$

*Joint H:*

$$g_H = \frac{2+2}{2+2+2+3} = 0.444$$

*Joint K:*

$$g_K = \frac{2+2}{2+2+3} = 0.571$$

***(2) Column factors:***

*Column factor, c = 1-g*

| ***Joint*** | ***g*** | ***c = (1-g)*** | ***g/2*** | ***c/2*** |
|---|---|---|---|---|
| *C* | *0.33* | *0.67* | *0.165* | *0.335* |
| *F* | *0.25* | *0.75* | *0.125* | *0.375* |
| *I* | *0.286* | *0.714* | *0.143* | *0.357* |
| *L* | *0.4* | *0.6* | *0.2* | *0.3* |
| *B* | *0.5* | *0.5* | *0.25* | *0.25* |

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| ***Joint*** | ***g*** | ***c = (1-g)*** | ***g/2*** | ***c/2*** |
|---|---|---|---|---|
| *E* | *0.4* | *0.6* | *0.2* | *0.3* |
| *H* | *0.444* | *0.556* | *0.222* | *0.278* |
| *K* | *0.571* | *0.429* | *0.285* | *0.215* |
| *A* | *0* | *1.0* | *0* | *0.5* |
| *D* | *0* | *1.0* | *0* | *0.5* |
| *G* | *0* | *1.0* | *0* | *0.5* |
| *J* | *0* | *1.0* | *0* | *0.5* |

***(3) Column and girder moment factors (C & G):***

| ***Joint*** | ***Members*** | ***c or g*** | ***Half values of factors of opposite end*** | ***(3) + (4)*** | ***k*** | ***C or G*** |
|---|---|---|---|---|---|---|
| *(1)* | *(2)* | *(3)* | *(4)* | *(5)* | *(6)* | *(5) * (6)* |
| *C* | *CF* | *0.33* | *0.125* | *0.455* | *4* | *1.82* |
| | *CB* | *0.67* | *0.25* | *0.92* | *2* | *1.84* |
| *F* | *FE* | *0.75* | *0.3* | *1.05* | *2* | *2.1* |
| | *FI* | *0.25* | *0.143* | *0.393* | *2* | *0.786* |
| | *FC* | *0.25* | *0.165* | *0.415* | *4* | *1.66* |
| *I* | *IF* | *0.286* | *0.125* | *0.411* | *2* | *0.822* |
| | *IH* | *0.714* | *0.278* | *0.992* | *2* | *1.984* |
| | *IL* | *0.286* | *0.2* | *0.486* | *3* | *1.458* |
| *L* | *LI* | *0.4* | *0.143* | *0.543* | *3* | *1.629* |
| | *LK* | *0.6* | *0.215* | *0.815* | *2* | *1.63* |
| *B* | *BE* | *0.5* | *0.2* | *0.7* | *4* | *2.8* |
| | *BC* | *0.5* | *0.335* | *0.835* | *2* | *1.67* |
| | *BA* | *0.5* | *0.5* | *1.0* | *2* | *2.0* |
| *E* | *EF* | *0.6* | *0.375* | *0.975* | *2* | *1.95* |
| | *EB* | *0.4* | *0.25* | *0.65* | *4* | *2.6* |
| | *EH* | *0.4* | *0.222* | *0.622* | *2* | *1.244* |
| | *ED* | *0.6* | *0.5* | *1.1* | *2* | *2.2* |

| **Structural Steel Design Project**<br><br>**Calculation Sheet** | Job No: | Sheet *4 of 8* | Rev |
|---|---|---|---|
| | Job Title: ***MULTI-STOREYED BUILDINGS*** | | |
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| | | Made by *SSSR* | Date *24-1-2000* |
| | | Checked by PU | Date *30-4-2000* |

| *Joint* | *Members* | *c or g* | *Half values of factors of opposite end* | *(3) + (4)* | *k* | *C or G* |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (5)* (6) |
| H | HI | 0.556 | 0.357 | 0.913 | 2 | 1.826 |
| | HE | 0.444 | 0.2 | 0.644 | 2 | 1.288 |
| | HG | 0.556 | 0.5 | 1.056 | 2 | 2.112 |
| | HK | 0.444 | 0.285 | 0.729 | 3 | 2.187 |
| K | KL | 0.429 | 0.3 | 0.729 | 2 | 1.458 |
| | KH | 0.571 | 0.222 | 0.793 | 3 | 2.379 |
| | KJ | 0.429 | 0.5 | 0.929 | 2 | 1.858 |
| A | AB | 1.0 | 0.25 | 1.25 | 2 | 2.5 |
| D | DE | 1.0 | 0.3 | 1.30 | 2 | 2.6 |
| G | GH | 1.0 | 0.278 | 1.278 | 2 | 2.556 |
| J | JK | 1.0 | 0.215 | 1.215 | 2 | 2.43 |

***(4) Storey Constants:***

*For ground storey,*

*Let $A_1$ be the storey constant for determination of moments at the ends of columns of the ground storey. Then*

*Total horizontal shear of ground storey = 15+20 = 35 kN*

*Height of ground storey = 6 m*

$$\sum C = (C_{AB} + C_{BA}) + (C_{ED} + C_{DE}) + (C_{HG} + C_{GH}) + (C_{KJ} + C_{JK})$$

$$= (2.5 + 2.0) + (2.2 + 2.6) + (2.112 + 2.556) + (1.858 + 2.43)$$

$$= 18.256$$

$$A_1 = \frac{35 \times 6}{18.256} = 11.5$$

| **Structural Steel Design Project** | Job No: | Sheet *5 of 8* | Rev |
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| | Job Title: ***MULTI-STOREYED BUILDINGS*** | | |
| | *Worked Example - 3* | | |
| | | Made by *SSSR* | Date *24-1-2000* |
| **Calculation Sheet** | | Checked by *PU* | Date *30-4-2000* |

*For top storey,*

*Let $A_2$ be the storey constant for determination of moments at the ends of columns of the top storey, then*

$$A_2 = \left( \frac{Total\ horizontal\ Shear\ of\ top\ storey\ \ X\ Height\ of\ top\ storey}{\Sigma C} \right)$$

*where, $\Sigma C$ = Sum of the column end moment factors of the storey.*

*Total horizontal shear of top storey = 15 kN.*

*Height of top storey = 4m.*

$\Sigma C = (C_{CB} + C_{BC}) + (C_{FE} + C_{EF}) + (C_{IH} + C_{HI}) + (C_{LK} + C_{KL})$

$= (1.84 + 1.67) + (2.1 + 1.95) + (1.984 + 1.826) + (1.63 + 1.458)$

$= 14.458$

$$A_2 = \frac{15 \times 4}{14.458} = 4.15$$

*(5) Moments at the ends of columns:*

*Ground storey moments:*

*Moment at end of the column = Column moment factor at that end *$A_1$*

$M_{AB} = 2.5 * 11.5 = 28.7$ kN-m $\qquad M_{BA} = 2.0 * 11.5 = 23.0$ kN-m

$M_{DE} = 2.6 * 11.5 = 29.9$ kN-m $\qquad M_{ED} = 2.2 * 11.5 = 25.3$ kN-m

$M_{GH} = 2.556 * 11.5 = 29.4$ kN-m $\qquad M_{HG} = 2.112 * 11.5 = 24.3$ kN-m

$M_{JK} = 2.43 * 11.5 = 27.9$ kN-m $\qquad M_{KJ} = 1.858 * 11.5 = 21.4$ kN-m

| **Structural Steel Design Project** | Job No: | Sheet *6 of 8* | Rev |
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*Top storey moments:*

*Moment at end of the column = Column moment factor at that end * $A_2$*

$M_{BC}$ = 1.67 * 4.15 = 6.93 kN-m    $M_{CB}$ = 1.84 * 4.15 = 7.64 kN-m

$M_{EF}$ = 1.95 * 4.15 = 8.09 kN-m    $M_{FE}$ = 2.1 * 4.15 = 8.72 kN-m

$M_{HI}$ = 1.826 * 4.15 = 7.58 kN-m    $M_{IH}$ = 1.984 * 4.15 = 8.23 kN-m

$M_{KL}$ = 1.458 * 4.15 = 6.05 kN-m    $M_{LK}$ = 1.63 * 4.15 = 6.76 kN-m

***(6) Joint Constants:***

*Joint constant (B)* **{** EMBED Equation.3 **}**

*For ground storey,*

$$B_B = \frac{M_{BC} + M_{BA}}{G_{BE}} = \frac{6.93 + 23.0}{2.8} = 10.69$$

$$B_E = \frac{M_{EF} + M_{ED}}{G_{EB} + G_{EH}} = \frac{8.09 + 25.3}{2.6 + 1.244} = 8.69$$

$$B_H = \frac{M_{HI} + M_{HG}}{G_{HE} + G_{HK}} = \frac{7.58 + 24.3}{1.288 + 2.187} = 9.17$$

$$B_K = \frac{M_{KL} + M_{KJ}}{G_{KH}} = \frac{6.05 + 21.37}{2.379} = 11.53$$

| Structural Steel Design Project<br><br>Calculation Sheet | Job No: | Sheet *7 of 8* | Rev |
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| | Job Title: ***MULTI-STOREYED BUILDINGS*** | | |
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*For top storey,*

$$B_C = \frac{M_{CB}}{G_{CF}} = \frac{7.64}{1.82} = 4.20$$

$$B_F = \frac{M_{FE}}{G_{FC} + G_{FI}} = \frac{8.72}{1.66 + 0.786} = 3.56$$

$$B_I = \frac{M_{IH}}{G_{IF} + G_{IL}} = \frac{8.23}{0.822 + 1.458} = 3.61$$

$$B_L = \frac{M_{LK}}{G_{LI}} = \frac{6.76}{1.629} = 4.15$$

***(7) Moments at the ends of beams:***

*Moment at the end of beam equals to Girder moment factor at that end multiplied by respective joint constant.*

$M_{CF}$ = 1.82 * 4.2 = 7.64 kN-m ; $M_{FC}$ = 1.66 * 3.56 = 5.91 kN-m

$M_{FI}$ = 0.786 * 3.56 = 2.8 kN-m ; $M_{IF}$ = 0.822* 3.61 = 2.97 kN-m

$M_{IL}$ = 1.458 * 3.61 = 5.26 kN-m ; $M_{LI}$ = 1.629 * 4.15 = 6.76 kN-m

$M_{BE}$ = 2.8 * 10.69 = 29.9 kN-m ; $M_{EB}$ = 2.6 * 8.69 = 22.6 kN-m

$M_{EH}$ = 1.244 * 8.69 = 10.8 kN-m ; $M_{HE}$ = 1.288 * 9.17 = 11.8 kN-m

$M_{HK}$ = 1.332 * 9.17 = 12.2 kN-m ; $M_{KH}$ = 2.187 * 11.53 = 25.2 kN-m

*The values of girder factors and column factors are shown in Fig. C.2*

| **Structural Steel Design Project** | Job No: | Sheet *8 of 8* | Rev |
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| | Job Title: ***MULTI-STOREYED BUILDINGS*** | | |
| | *Worked Example - 3* | | |
| | | Made by *SSSR* | Date *24-1-2000* |
| **Calculation Sheet** | | Checked by *PU* | Date *30-4-2000* |

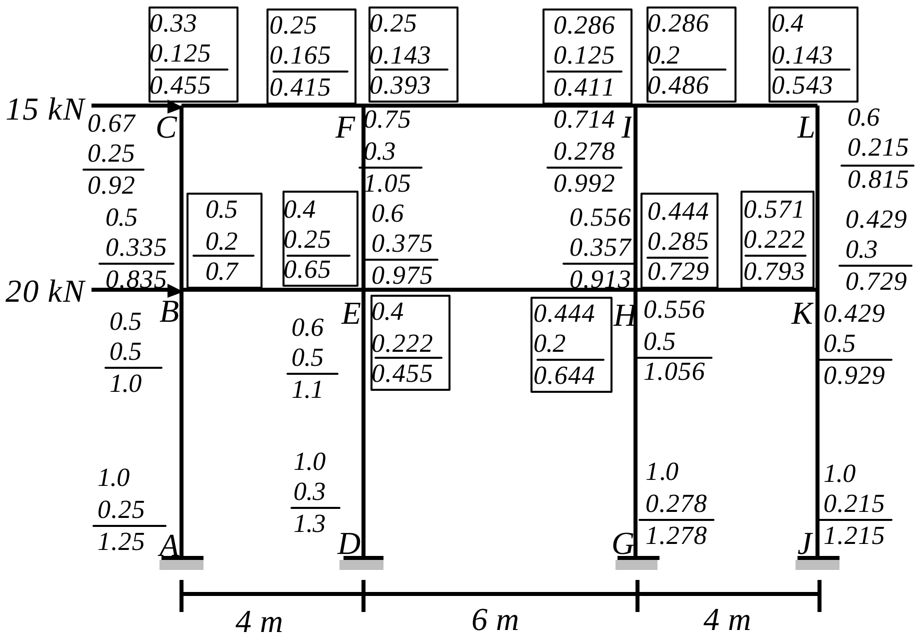



***Fig. C. 2 Girder factors and column factors of the frame***

*Factors presented in rectangular boxes are girder factors and column factors are presented simply without any rectangular box.*

| **Structural Steel Design Project** <br><br> **Calculation Sheet** | Job No: | Sheet ***1 of 2*** | Rev |
|---|---|---|---|
| | Job Title: ***MULTI-STOREYED BUILDINGS*** | | |
| | *Worked Example - 4* | | |
| | | Made by *SSSR* | Date *24-1-2000* |
| | | Checked by *PU* | Date *30-4-2000* |

*Problem 4:*

*Determine the moments at the ends of columns and beams of the rigidly jointed building frame shown in Fig. D for the gravity load applied.*

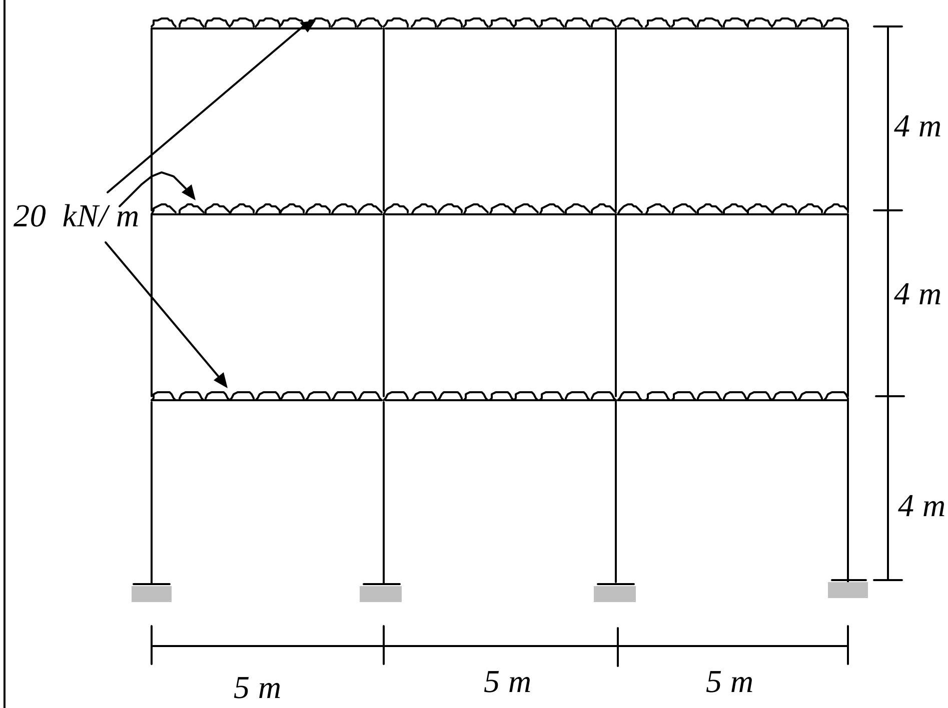
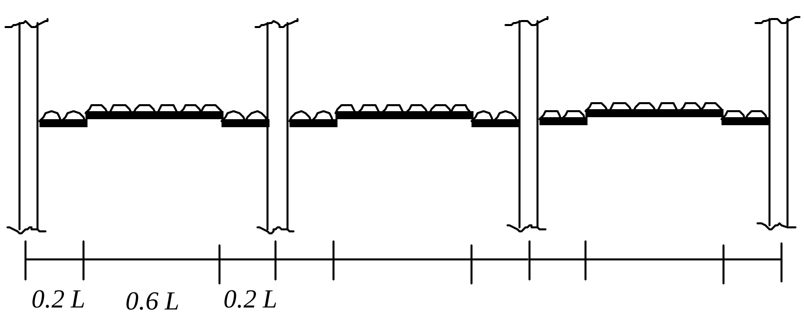



*Fig. D*

*Consider the following approximate model.*




*L = 5 m* ***Approximate model***

| **Structural Steel Design Project** | Job No: | Sheet *2 of 2* | Rev |
|---|---|---|---|
| | Job Title: ***MULTI-STOREYED BUILDINGS*** | | |
| | *Worked Example - 4* | | |
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| **Calculation Sheet** | | Checked by *PU* | Date *30-4-2000* |

*Maximum + ve B.M. at mid-span* = $\frac{wL^2}{8}$

$= 20 * 3^2/8 = 22.5$ *kN-m*

*End reaction* = *wl/2* $= 20 * 3/2 = 30$ *kN*

*Maximum negative B.M. at end column* = $30 * 1 + (20 * 1 * 1) / 2$

$= 40$ *kN-m*

*Bending moment in the interior column* = $40 – 40 = 0$

***B.M. diagram for the frame:***