

## 1.0 INTRODUCTION

In a previous chapter, the design of a steel-concrete composite column under axial loading was discussed. This chapter deals with the design of steel-concrete composite columns subjected to both axial load and bending. To design a composite column under combined compression and bending, it is first isolated from the framework, and the end moments which result from the analysis of the system as a whole are taken to act on the column under consideration. Internal moments and forces within the column length are determined from the structural consideration of end moments, axial and transverse loads. For each axis of symmetry, the buckling resistance to compression is first checked with the relevant non-dimensional slenderness of the composite column. Thereafter the moment resistance of the composite cross-section is checked in the presence of applied moment about each axis, e.g.  $x-x$  and  $y-y$  axis, with the relevant non-dimensional slenderness values of the composite column. For slender columns, both the effects of long term loading and the second order effects are included.

## 2.0 COMBINED COMPRESSION AND UNI-AXIAL BENDING

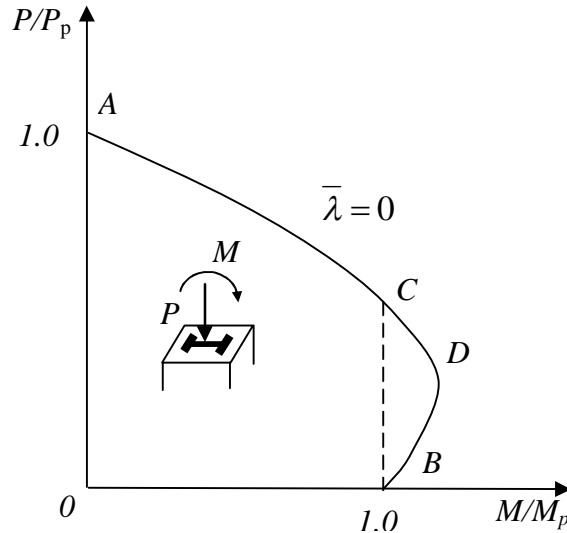
The design method described here is an extension of the simplified design method discussed in the previous chapter for the design of steel-concrete composite columns under axial load.

### 2.1 Interaction Curve for Compression and Uni-axial Bending

The resistance of the composite column to combined compression and bending is determined using an interaction curve. Fig. 1 represents the non-dimensional interaction curve for compression and uni-axial bending for a composite cross-section.

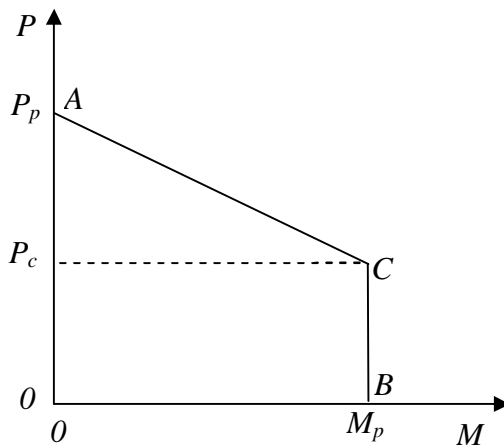
In a typical interaction curve of a column with steel section only, it is observed that the moment of resistance undergoes a continuous reduction with an increase in the axial load. However, a short composite column will often exhibit increases in the moment resistance beyond plastic moment under relatively low values of axial load. This is because under some favourable conditions, the compressive axial load would prevent concrete cracking and make the composite cross-section of a short column more effective in resisting moments. The interaction curve for a short composite column can be obtained by considering several positions of the neutral axis of the cross-section,  $h_n$ , and determining the internal forces and moments from the resulting stress blocks.

(It should be noted by way of contrast that *IS: 456-1978* for reinforced concrete columns specifies a 2 cm eccentricity irrespective of column geometry. The method suggested here, using EC4, allows for an eccentricity of load application by the term  $\alpha$  and therefore no further provision is necessary for steel columns. Another noteworthy feature is the prescription of strain limitation in *IS: 456-1978*, whereas EC4 does not impose such a limitation. The relevant provision in the Indian Code limits the concrete strain to 0.0035 minus 0.75 times the strain at the least compressed extreme fibre)



**Fig. 1 Interaction curve for compression and uni-axial bending**

Fig. 2 shows an interaction curve drawn using simplified design method suggested in the UK National Application Document for EC 4 (*NAD*). This neglects the increase in moment capacity beyond  $M_p$  discussed above, (under relatively low axial compressive loads).



**Fig. 2 Interaction curve for compression and uni-axial bending using the simplified method**

Fig. 3 shows the stress distributions in the cross-section of a concrete filled rectangular tubular section at each point, *A*, *B* and *C* of the interaction curve given in Fig. 2. It is important to note that:

- Point *A* marks the plastic resistance of the cross-section to compression (at this point the bending moment is zero).

$$P_A = P_p = A_a f_y / \gamma_a + \alpha_c A_c (f_{ck})_{cy} / \gamma_c + A_s f_{sk} / \gamma_s \quad (1)$$

$$M_A = 0 \quad (2)$$

- Point *B* corresponds to the plastic moment resistance of the cross-section (the axial compression is zero).

$$P_B = 0 \quad (3)$$

$$M_B = M_p = p_y (Z_{pa} - Z_{pan}) + p_{sk} (Z_{ps} - Z_{psn}) + p_{ck} (Z_{pc} - Z_{pcn}) \quad (4)$$

where

$Z_{ps}$ ,  $Z_{pa}$ , and  $Z_{pc}$  are plastic section moduli of the reinforcement, steel section, and concrete about their own centroids respectively.

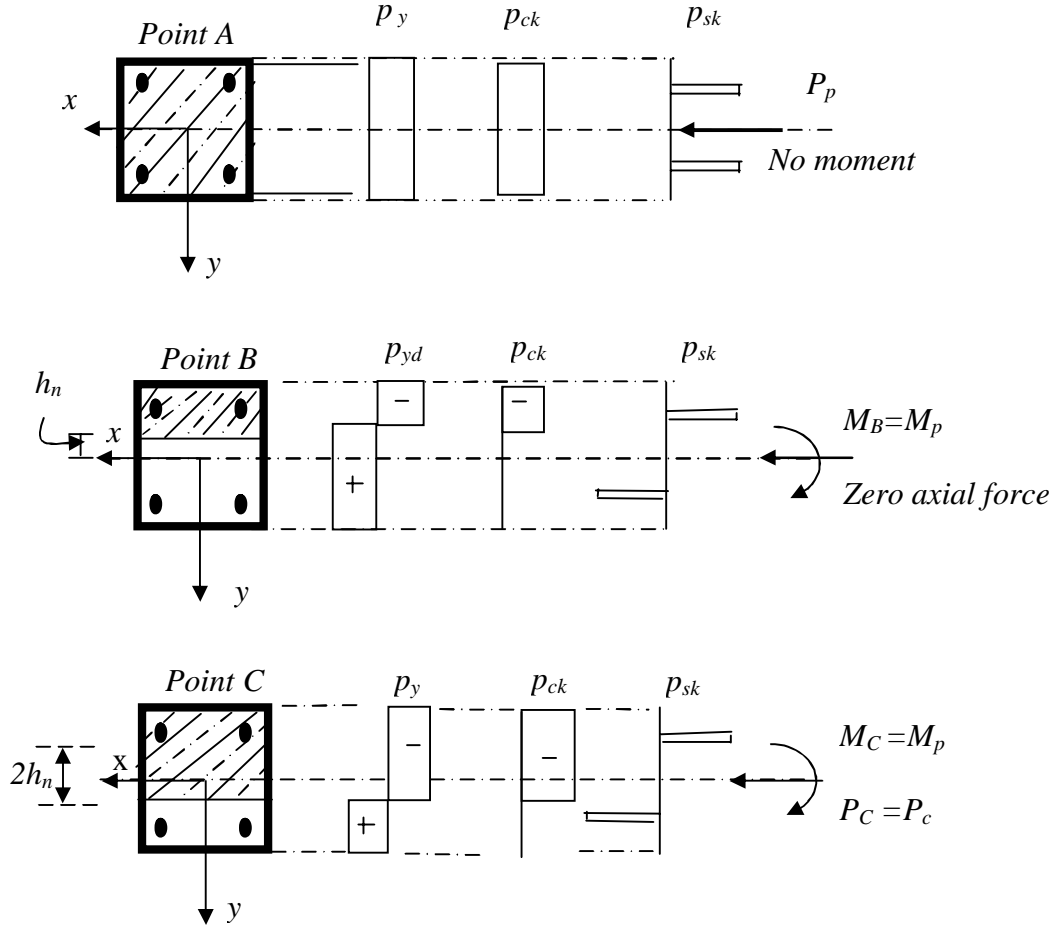
$Z_{psn}$ ,  $Z_{pan}$  and  $Z_{pcn}$  are plastic section moduli of the reinforcement, steel section, and concrete about neutral axis respectively.

- At point *C*, the compressive and the moment resistances of the column are given as follows;

$$P_C = P_c = A_c p_{ck} \quad (5)$$

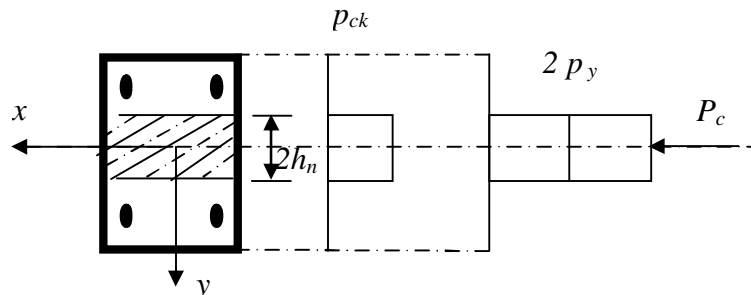
$$M_C = M_p \quad (6)$$

- The expressions may be obtained by combining the stress distributions of the cross-section at points *B* and *C*; the compression area of the concrete at point *B* is equal to the tension area of the concrete at point *C*. The moment resistance at point *C* is equal to that at point *B*, since the stress resultants from the additionally compressed parts nullify each other in the central region of the cross-section. However, these additionally compressed regions create an internal axial force, which is equal to the plastic resistance to compression of the concrete,  $P_c$  alone.



**Fig. 3 Stress distributions for the points of the interaction curve for concrete filled rectangular tubular sections**

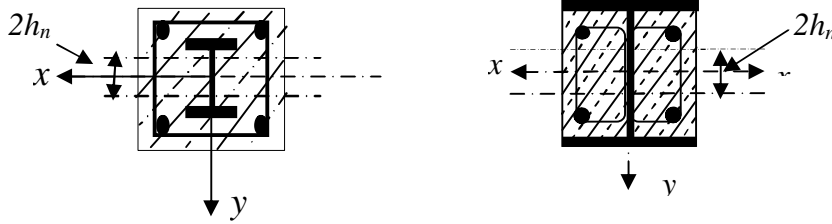
It is important to note that the positions of the neutral axis for points B and C,  $h_n$ , can be determined from the difference in stresses at points B and C. The resulting axial forces, which are dependent on the position of the neutral axis of the cross-section,  $h_n$ , can easily be determined as shown in Fig. 4. The sum of these forces is equal to  $P_c$ . This calculation enables the equation defining  $h_n$  to be determined, which is different for various types of sections.



**Fig. 4(a) Variation in the neutral axis positions**

(1) For concrete encased steel sections:

**Major axis bending**



**Fig. 4(b)**

(1) Neutral axis in the web:  $h_n \leq [h/2 - t_f]$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w (2p_y - p_{ck})}$$

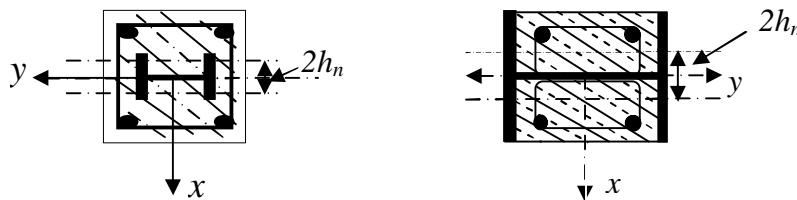
(2) Neutral axis in the flange:  $[h/2 - t_f] \leq h_n \leq h/2$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck}) + (b - t_w)(h - 2t_f)(2p_y - p_{ck})}{2b_c p_{ck} + 2b(2p_y - p_{ck})}$$

(3) Neutral axis outside the steel section:  $h/2 \leq h_n \leq h_c/2$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck}) + A_a (2p_y - p_{ck})}{2b_c p_{ck}}$$

**Minor axis bending**



**Fig. 4(c)**

(1) Neutral axis in the web:  $h_n \leq t_w/2$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck})}{2h_c p_{ck} + 2h(2p_y - p_{ck})}$$

(2) Neutral axis in the flange:  $t_w/2 < h_n < b/2$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck}) + t_w (2t_f - h)(2p_y - p_{ck})}{2h_c p_{ck} + 4t_f (2p_y - p_{ck})}$$

(3) Neutral axis outside the steel section:  $b/2 \leq h_n \leq b_c/2$

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck}) - A_a (2p_y - p_{ck})}{2h_c p_{ck}}$$

Note:  $A'_s$  is the sum of the reinforcement area within the region of  $2h_n$

(2) For concrete filled tubular sections

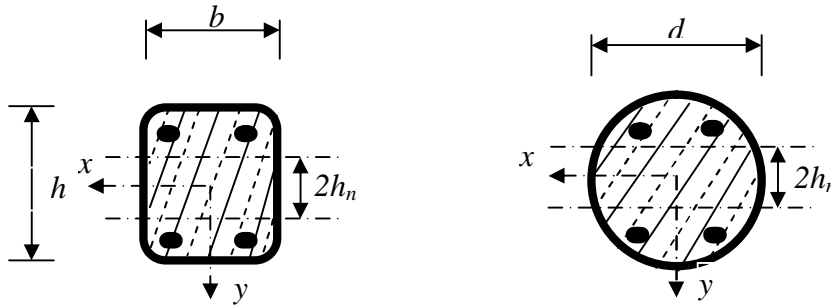


Fig. 4(d)

**Major axis bending**

$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck})}{2b_c p_{ck} + 4t(2p_y - p_{ck})}$$

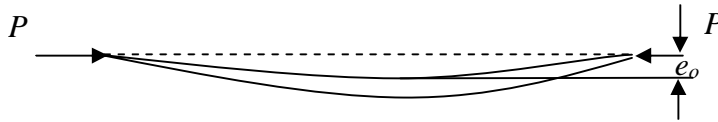
Note:

- For circular tubular section substitute  $b_c = d$
- For minor axis bending the same equations can be used by interchanging  $h$  and  $b$  as well as the subscripts  $x$  and  $y$ .

## 2.2 Analysis of Bending Moments due to Second Order Effects

Under the action of a design axial load,  $P$ , on a column with an initial imperfection,  $e_o$ , as shown in Fig. 5, there will be a maximum internal moment of  $P.e_o$ . It is important to note that this second order moment, or 'imperfection moment', does not need to be considered separately, as its effect on the buckling resistance of the composite column is already accounted for in the European buckling curves.

However, in addition to axial forces, a composite column may be also subject to end moments as a consequence of transverse loads acting on it, or because the composite column is a part of a frame. The moments and the displacements obtained initially are referred to as ‘first order’ values. For slender columns, the ‘first order’ displacements may be significant and additional or ‘second order’ bending moments may be induced under the actions of applied loads. As a simple rule, the second order effects should be considered if the buckling length to depth ratio of a composite column exceeds 15.



**Fig. 5 Initially imperfect column under axial compression**

The second order effects on bending moments for isolated non-sway columns should be considered if both of the following conditions are satisfied:

$$(1) \quad \frac{P}{P_{cr}} > 0.1 \quad (7)$$

where

$P$  is the design applied load, and

$P_{cr}$  is the elastic critical load of the composite column.

(2) Elastic slenderness conforms to:

$$\bar{\lambda} > 0.2 \quad (8)$$

where

$\bar{\lambda}$  is the non-dimensional slenderness of the composite column

In case the above two conditions are met, the second order effects may be allowed for by modifying the maximum first order bending moment (moment obtained initially),  $M_{max}$ , with a correction factor  $k$ , which is defined as follows:

$$k = \frac{1}{1 - \frac{P}{P_{cr}}} \geq 1.0 \quad (9)$$

where

$P$  is the applied design load.

$P_{cr}$  is the elastic critical load of the composite column.

### 2.3 Resistance of Members under Combined Compression and Uni-axial Bending

The graphical representation of the principle for checking the composite cross-section under combined compression and uni-axial bending is illustrated in Fig. 6.

The design checks are carried out in the following stages:

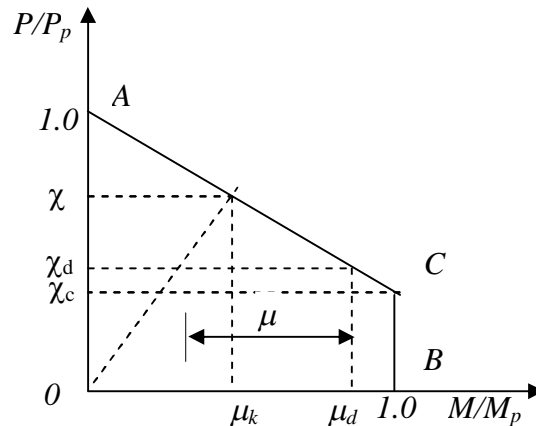
- (1) The resistance of the composite column under axial load is determined in the absence of bending, which is given by  $\chi P_p$ . The procedure is explained in detail in the previous chapter.
- (2) The moment resistance of the composite column is then checked with the relevant non-dimensional slenderness, in the plane of the applied moment. As mentioned before, the initial imperfections of columns have been incorporated and no additional consideration of geometrical imperfections is necessary.

The design is adequate when the following condition is satisfied:

$$M \leq 0.9\mu M_p \quad (10)$$

where

- $M$  is the design bending moment, which may be factored to allow for second order effects, if necessary
- $\mu$  is the moment resistance ratio obtained from the interaction curve.
- $M_p$  is the plastic moment resistance of the composite cross-section.



**Fig. 6 Interaction curve for compression and uni-axial bending using the simplified method**

The interaction curve shown in Fig. 6 has been determined without considering the strain limitations in the concrete. Hence the moments, including second order effects if necessary, are calculated using the effective elastic flexural stiffness,  $(EI)_e$ , and taking into account the entire concrete area of the cross-section, (i.e. concrete is uncracked).



Consequently, a reduction factor of 0.9 is applied to the moment resistance as shown in Equation (10) to allow for the simplifications in this approach. If the bending moment and the applied load are independent of each other, the value of  $\mu$  must be limited to 1.0.

Moment resistance ratio  $\mu$  can be obtained from the interaction curve or may be evaluated. The method is described below.

Consider the interaction curve for combined compression and bending shown in Fig. 6. Under an applied force  $P$  equal to  $\chi P_p$ , the horizontal coordinate  $\mu_k M_p$  represents the second order moment due to imperfections of the column, or the 'imperfection moment'. It is important to recognise that the moment resistance of the column has been fully utilised in the presence of the 'imperfection moment'; the column cannot resist any additional applied moment.

$\chi_d$  represents the axial load ratio defined as follows:

$$\chi_d = \frac{P}{P_p} \quad (11)$$

By reading off the horizontal distance from the interaction curve, the moment resistance ratio,  $\mu$ , may be obtained and the moment resistance of the composite column under combined compression and bending may then be evaluated.

In accordance with the *UK NAD*, the moment resistance ratio  $\mu$  for a composite column under combined compression and uni-axial bending is evaluated as follows:

$$\mu = \frac{(\chi - \chi_d)}{(1 - \chi_c)\chi} \quad \text{when } \chi_d \geq \chi_c \quad (12)$$

$$= 1 - \frac{(1 - \chi)\chi_d}{(1 - \chi_c)\chi} \quad \text{when } \chi_d < \chi_c \quad (13)$$

where

$\chi_c$  = axial resistance ratio due to the concrete,  $\frac{P_c}{P_p}$

$\chi_d$  = design axial resistance ratio,  $\frac{P}{P_p}$

$\chi$  = reduction factor due to column buckling

The expression is obtained from geometry consideration of the simplified interaction curve illustrated in Fig. 6. A worked example illustrating the use of the above design procedure is appended to this chapter.

### 3.0 COMBINED COMPRESSION AND BI-AXIAL BENDING

For the design of a composite column under combined compression and bi-axial bending, the axial resistance of the column in the presence of bending moment for each axis has to be evaluated separately. Thereafter the moment resistance of the composite column is checked in the presence of applied moment about each axis, with the relevant non-dimensional slenderness of the composite column. Imperfections have to be considered only for that axis along which the failure is more likely. If it is not evident which plane is more critical, checks should be made for both the axes.

The moment resistance ratios  $\mu_x$  and  $\mu_y$  for both the axes are evaluated as given below:

$$\mu_x = \frac{(\chi_x - \chi_d)}{(1 - \chi_c)\chi_x} \quad \text{when } \chi_d \geq \chi_c \quad (14)$$

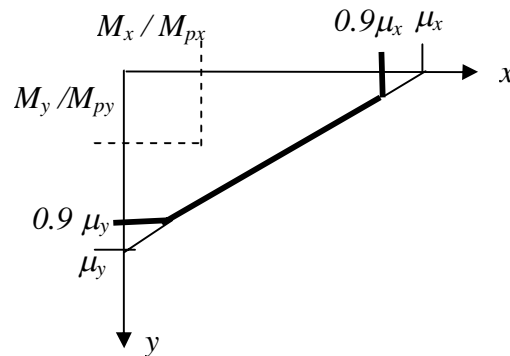
$$= 1 - \frac{(1 - \chi_x)\chi_d}{(1 - \chi_c)\chi_x} \quad \text{when } \chi_d < \chi_c \quad (15)$$

$$\mu_y = \frac{(\chi_y - \chi_d)}{(1 - \chi_c)\chi_y} \quad \text{when } \chi_d \geq \chi_c \quad (16)$$

$$= 1 - \frac{(1 - \chi_y)\chi_d}{(1 - \chi_c)\chi_y} \quad \text{when } \chi_d < \chi_c \quad (17)$$

where

$\chi_x$  and  $\chi_y$  are the reduction factors for buckling in the  $x$  and  $y$  directions respectively.



**Fig. 7 Moment interaction curve for bi-axial bending**

In addition to the two conditions given by Equations (18) and (19), the interaction of the moments must also be checked using moment interaction curve as shown in Fig. 7. The linear interaction curve is cut off at  $0.9\mu_x$  and  $0.9\mu_y$ . The design moments,  $M_x$  and  $M_y$  related to the respective plastic moment resistances must lie within the moment interaction curve.

Hence the three conditions to be satisfied are:

$$\frac{M_x}{\mu_x M_{px}} \leq 0.9 \quad (18)$$

$$\frac{M_y}{\mu_y M_{py}} \leq 0.9 \quad (19)$$

$$\frac{M_x}{\mu_x M_{px}} + \frac{M_y}{\mu_y M_{py}} \leq 1.0 \quad (20)$$

When the effect of geometric imperfections is not considered the moment resistance ratio is evaluated as given below:

$$\mu = \frac{(1 - \chi_d)}{(1 - \chi_c)} \quad \text{when } \chi_d > \chi_c \quad (21)$$

$$= 1.0 \quad \text{when } \chi_d \leq \chi_c \quad (22)$$

A worked example on combined compression and bi-axial bending is appended to this chapter.

## 4.0 STEPS IN DESIGN

### 4.1 Design Steps for columns with axial load and uni-axial bending

4.1.1 List the composite column specifications and the design values of forces and moments.

4.1.2 List material properties such as  $f_y$ ,  $f_{sk}$ ,  $(f_{ck})_{cy}$ ,  $E_a$ ,  $E_s$ ,  $E_c$

4.1.3 List section properties  $A_a$ ,  $A_s$ ,  $A_c$ ,  $I_a$ ,  $I_s$ ,  $I_c$  of the selected section

4.1.4 Design checks

(1) Evaluate plastic resistance,  $P_p$  of the cross-section from equation,

$$P_p = A_a f_y / \gamma_a + \alpha_c A_c (f_{ck})_{cy} / \gamma_c + A_s f_{sk} / \gamma_s$$

(2) Evaluate effective flexural stiffness,  $(EI)_e$  of the cross-section for short term loading in  $x$  and  $y$  direction using equation,

$$(EI)_e = E_a I_a + 0.8 E_{cd} I_c + E_s I_s$$

(3) Evaluate non-dimensional slenderness,  $\bar{\lambda}_x$  and  $\bar{\lambda}_y$  in  $x$  and  $y$  directions from equation,

$$\bar{\lambda} = \left( \frac{P_{pu}}{P_{cr}} \right)^{\frac{1}{2}}$$

where

$$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$$

Note:  $P_{pu}$  is the plastic resistance of the section with  $\gamma_a = \gamma_c = \gamma_s = 1.0$

$$\text{and } P_{cr} = \frac{\pi^2 (EI)_e}{\ell^2}$$

(4) Check for long-term loading

The effect of long term loading can be neglected if following conditions are satisfied:

- Eccentricity,  $e$  given by

$$e = M/P \geq 2 \text{ times the cross section dimension in the plane of bending considered}$$

- the non-dimensional slenderness  $\bar{\lambda}$  in the plane of bending being considered exceeds the limits given in Table 6 of the previous chapter ( Steel Concrete Composite Column-I)

(5) Check the resistance of the section under axial compression for both  $x$  and  $y$  axes.

Design against axial compression is satisfied if following condition is satisfied for both the axes:

$$P < \chi P_p$$

where

$\chi$  = reduction factor due to column buckling.

$$\chi = \frac{1}{\left( \phi + \left\{ \phi^2 - \bar{\lambda}^2 \right\}^{\frac{1}{2}} \right)}$$

where

$$\text{and } \phi = 0.5 \left[ 1 + \alpha (\bar{\lambda} - 0.2) + \bar{\lambda}^2 \right]$$

(6) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if following conditions are satisfied for the plane of bending under consideration:

- $P / P_{cr} \leq 0.1$
- $\bar{\lambda} \leq 0.2$

(7) Evaluate plastic moment resistance of the composite column about the plane of bending under consideration.

$$M_p = p_y (Z_{pa} - Z_{pan}) + 0.5 p_{ck} (Z_{pc} - Z_{pcn}) + p_{sk} (Z_{ps} - Z_{psn})$$

where

$Z_{ps}$ ,  $Z_{pa}$ , and  $Z_{pc}$  are plastic section modulus of the reinforcement, steel section, and concrete about their own axes respectively.

$Z_{psn}$ ,  $Z_{pan}$ , and  $Z_{pcn}$  are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis respectively.

(8) Check the resistance of the composite column under combined axial compression and uni-axial bending

The design against combined compression and uni-axial bending is adequate if following condition is satisfied:

$$M \leq 0.9 \mu M_p$$

where

- $M$  design bending moment
- $M_p$  plastic moment resistance
- $\mu$  moment resistance ratio

## 4.2 Design Steps for columns with axial load and bi-axial bending

4.2.1 List the composite column specifications and the design values of forces and

moments.

4.2.2 List material properties such as  $f_y$ ,  $f_{sk}$ ,  $(f_{ck})_{cy}$ ,  $E_a$ ,  $E_s$ ,  $E_c$

4.2.3 List section properties  $A_a$ ,  $A_s$ ,  $A_c$ ,  $I_a$ ,  $I_s$ ,  $I_c$  of the selected section.

4.2.4 Design checks

(1) Evaluate plastic resistance,  $P_p$  of the cross-section from equation,

$$P_p = A_a f_y / \gamma_a + \alpha_c A_c (f_{ck})_{cy} / \gamma_c + A_s f_{sk} / \gamma_s$$

(2) Evaluate effective flexural stiffness,  $(EI)_{ex}$  and  $(EI)_{ey}$ , of the cross-section for short term loading from equation,

$$(EI)_{ex} = E_a I_{ax} + 0.8 E_{cd} I_{cx} + E_s I_{sx}$$

$$(EI)_{ey} = E_a I_{ay} + 0.8 E_{cd} I_{cy} + E_s I_{sy}$$

(3) Evaluate non-dimensional slenderness,  $\bar{\lambda}_x$  and  $\bar{\lambda}_y$  from equation,

$$\bar{\lambda}_x = \left( \frac{P_{pu}}{(P_{cr})_x} \right)^{\frac{1}{2}}$$

$$\bar{\lambda}_y = \left( \frac{P_{pu}}{(P_{cr})_y} \right)^{\frac{1}{2}}$$

where

$$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$$

Note:  $P_{pu}$  is the plastic resistance of the section with  $\gamma_a = \gamma_c = \gamma_s = 1.0$

$$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$$

$$\text{and } (P_{cr})_y = \frac{\pi^2 (EI)_{ey}}{\ell^2}$$

(4) Check for long term loading.

The effect of long-term loading can be neglected if following conditions are satisfied:

- Eccentricity,  $e$  given by

$e = M/P \geq 2$  times cross section dimension in the plane of bending considered.

$$e_x \geq 2b_c$$

$$\text{and } e_y \geq 2h_c$$

- the non-dimensional slenderness  $\bar{\lambda}$  in the plane of bending being considered exceeds the limits given in Table 6 of the previous chapter ( Steel Concrete Composite Column –I).

(5) Check the resistance of the section under axial compression about both the axes. Design against axial compression is satisfied if following conditions are satisfied:

$$P < \chi_x P_p$$

$$P < \chi_y P_p$$

where

$$\chi_x = \frac{1}{\left( \phi_x + \left\{ \phi_x^2 - \bar{\lambda}_x^2 \right\}^{\frac{1}{2}} \right)}$$

$$\text{and } \phi_x = 0.5 \left[ 1 + \alpha_x (\bar{\lambda}_x - 0.2) + \bar{\lambda}_x^2 \right]$$

$$\chi_y = \frac{1}{\left( \phi_y + \left\{ \phi_y^2 - \bar{\lambda}_y^2 \right\}^{\frac{1}{2}} \right)}$$

$$\text{and } \phi_y = 0.5 \left[ 1 + \alpha_y (\bar{\lambda}_y - 0.2) + \bar{\lambda}_y^2 \right]$$

(6) Check for second order effects

Isolated non – sway columns need not be checked for second order effects if:

$$P / (P_{cr})_x \leq 0.1 \quad \text{for bending about } x\text{-}x \text{ axis}$$

$$P / (P_{cr})_y \leq 0.1 \quad \text{for bending about } y\text{-}y \text{ axis}$$

(7) Evaluate plastic moment resistance of the composite column under axial compression and bi-axial bending about both the axes.

About x-x axis

$$M_{px} = [p_y (Z_{pa} - Z_{pan}) + 0.5 p_{ck} (Z_{pc} - Z_{pcn}) + p_{sk} (Z_{ps} - Z_{psn})] I_x$$

where

$M_{px}$  plastic moment resistance about x-x axis

$Z_{psx}$ ,  $Z_{pax}$ , and  $Z_{pcx}$  are plastic section modulus of the reinforcement, steel section, and concrete about their own axes in x direction respectively.

$Z_{psn}$ ,  $Z_{pan}$ , and  $Z_{pcn}$  are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis in x direction respectively.

About y-y axis

$$M_{py} = [p_y (Z_{pay} - Z_{pan}) + 0.5 p_{ck} (Z_{pcy} - Z_{pcn}) + p_{sk} (Z_{psy} - Z_{psn})] I_y$$

where

$M_{py}$  plastic moment resistance about y-y axis

$Z_{psy}$ ,  $Z_{pay}$ , and  $Z_{pcy}$  are plastic section moduli of the reinforcement, steel section, and concrete about their own axes in y direction respectively.

$Z_{psn}$ ,  $Z_{pan}$ , and  $Z_{pcn}$  are plastic section modulus of the reinforcement, steel section, and concrete about neutral axis in y direction respectively.

(8) Evaluate resistance of the composite column under combined axial compression and bi-axial bending

The design against combined compression and bi-axial bending is adequate if following conditions are satisfied:

$$(1) M_x \leq 0.9 \mu_x M_{Px}$$

$$(2) M_y \leq 0.9 \mu_y M_{Py}$$

$$(3) \frac{M_x}{\mu_x M_{Px}} + \frac{M_y}{\mu_y M_{Py}} \leq 1.0$$

where

$\mu_x$  and  $\mu_y$  are the moment resistance ratios in the x and y directions respectively.



## 5.0 CONCLUSION

In this chapter the design of steel-concrete composite column subjected to axial load and bending is discussed. The use of interaction curve in the design of composite column subjected to both uni-axial bending and bi-axial bending is also described. Worked out example in each case is also appended to this chapter.

## NOTATION

$A$	cross-sectional area
$b$	breadth of element
$d$	diameter, depth of element.
$e$	eccentricity of loading
$e_o$	initial imperfections
$E$	modulus of elasticity
$(EI)_e$	effective elastic flexural stiffness of a composite cross-section.
$(f_{ck})_{cu}$	characteristic compressive (cube) strength of concrete
$(f_{ck})_{cy}$	characteristic compressive (cylinder) strength of concrete, given by 0.80 times 28 days cube strength of concrete.
$f_{sk}$	characteristic strength of reinforcement
$f_y$	yield strength of steel
$f_{ctm}$	mean tensile strength of concrete
$p_{ck}, p_y, p_{sk}$	design strength of concrete, steel section and reinforcement respectively
$h$	height of element
$h_n$	depth of neutral axis from the middle line of the cross-section
$I$	second moment of area (with subscripts)
$k$	moment correction factor for second order effects
$\ell$	buckling (or effective) length
$L$	length or span
$M$	moment (with subscripts)
$P$	axial force
$M_p$	plastic moment resistance of a cross-section
$P_p$	plastic resistance to compression of the cross section.
$P_{pu}$	plastic resistance to compression of the cross section with $\gamma_a = \gamma_c = \gamma_s = 1.0$
$P_{cr}$	elastic critical load of a column
$P_c$	axial resistance of concrete, $A_c p_{ck}$
$t$	thickness of element
$Z_p$	plastic section modulus

## Greek letters

$\gamma_f$	partial safety factor for loads
$\gamma$	partial safety factor for materials (with subscripts)
$\gamma_c^*$	Reduction factor(1.35) used for reducing $E_{cm}$ value
$\lambda$	slenderness ( $\bar{\lambda}$ = non-dimensional slenderness)
$\varepsilon$	coefficient $\sqrt{250/f_y}$
$\alpha$	imperfection factor
$\alpha_c$	strength coefficient for concrete
$\chi$	reduction factor buckling
$\chi_c$	axial resistance ratio due to concrete, $P_c/P_p$
$\mu$	moment resistance ratio

The subscripts to the above symbols are as follows:

$a$	structural steel
$b$	buckling
$c$	concrete
$f$	flange
$k$	characteristic value
$s$	reinforcement
$w$	web of steel section

Note-The subscript  $x, y$  denote the  $x-x$  and  $y-y$  axes of the section respectively.  $x-x$  denotes the major axes whilst  $y-y$  denotes the minor principal axes.

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	Job Title: <i>Design of Composite Column with Axial load and Uni-axial bending</i>															
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	<p><u><b>PROBLEM 1</b></u></p> <p><i>Check the adequacy of the concrete encased composite section shown below for uni-axial bending.</i></p> <div style="text-align: center;"> <p>The diagram shows a square cross-section of a column with a side length of 350 mm. Inside the square is an ISHB 250 steel section. Four reinforcement bars of 14 mm diameter are shown at the corners of the square. The x-x and y-y axes are indicated.</p> </div> <p><b>4.1.1 DETAILS OF THE SECTION</b></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 40%;"><i>Column dimension</i></td> <td style="width: 60%;">350 X 350 X 3000</td> </tr> <tr> <td><i>Concrete Grade</i></td> <td>M30</td> </tr> <tr> <td><i>Steel Section</i></td> <td>ISHB 250</td> </tr> <tr> <td><i>Steel Reinforcement</i></td> <td>4 Nos. of 14 mm dia bar, Fe415 grade</td> </tr> <tr> <td><i>Design Axial Load</i></td> <td>1500 kN</td> </tr> <tr> <td><i>Design bending moment about x-x axis</i></td> <td>180 kNm</td> </tr> <tr> <td><i>Design bending moment about y-y axis</i></td> <td>0 kNm</td> </tr> </table>			<i>Column dimension</i>	350 X 350 X 3000	<i>Concrete Grade</i>	M30	<i>Steel Section</i>	ISHB 250	<i>Steel Reinforcement</i>	4 Nos. of 14 mm dia bar, Fe415 grade	<i>Design Axial Load</i>	1500 kN	<i>Design bending moment about x-x axis</i>	180 kNm	<i>Design bending moment about y-y axis</i>
<i>Column dimension</i>	350 X 350 X 3000															
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<i>Design bending moment about y-y axis</i>	0 kNm															
			<p><i>Axial Load</i> <math>P = 1500 \text{ kN}</math></p> <p><math>M_x = 180 \text{ kN}</math></p>													

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<p><b><u>DESIGN CALCULATIONS:</u></b></p> <p><b>4.1.2 LIST MATERIAL PROPERTIES</b></p> <p><b>(1) Structural steel</b></p> <p>Steel section ISHB 250 Nominal yield strength <math>f_y = 250 \text{ N/mm}^2</math> Modulus of elasticity <math>E_a = 200 \text{ kN/mm}^2</math></p> <p><b>(2) Concrete</b></p> <p>Concrete grade M30 Characteristic strength <math>(f_{ck})_{cu} = 30 \text{ N/mm}^2</math> Secant modulus of elasticity for short term loading, <math>E_{cm} = 31220 \text{ N/mm}^2</math></p> <p><b>(3) Reinforcing steel</b></p> <p>Steel grade Fe 415 Characteristic strength <math>f_{sk} = 415 \text{ N/mm}^2</math> Modulus of elasticity <math>E_s = 200 \text{ kN/mm}^2</math></p> <p><b>(4) Partial safety factors</b></p> <p><math>\gamma_a = 1.15</math> <math>\gamma_c = 1.5</math> <math>\gamma_s = 1.15</math></p> <p><b>4.1.3 SECTION PROPERTIES OF THE GIVEN SECTION</b></p> <p><b>(1) Steel section</b></p> <p><math>A_a = 6971 \text{ mm}^2</math></p>			

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<p> <math>t_f = 9.7 \text{ mm}; h = 250 \text{ mm}; t_w = 8.8 \text{ mm}</math>  <math>I_{ax} = 79.8 * 10^6 \text{ mm}^4</math>  <math>I_{ay} = 20.1 * 10^6 \text{ mm}^4</math>  <math>Z_{pax} = 699.8 * 10^3 \text{ mm}^3</math>  <math>Z_{pay} = 307.6 * 10^3 \text{ mm}^3</math> </p> <p><b>(2) Reinforcing steel</b></p> <p>4 bars of 14 mm dia, <math>A_s = 616 \text{ mm}^2</math></p> <p><b>(3) Concrete</b></p> $A_c = A_{gross} - A_a - A_s$ $= 350 * 350 - 6971 - 616$ $= 114913 \text{ mm}^2$ <p><b>4.1.4 DESIGN CHECKS</b></p> <p><b>(1) Plastic resistance of the section</b></p> $P_p = A_a f_y / \gamma_a + \alpha_c A_c (f_{ck})_{cy} / \gamma_c + A_s f_{sk} / \gamma_s$ $P_p = A_a f_y / \gamma_a + \alpha_c A_c (0.80 * (f_{ck})_{cu}) / \gamma_c + A_s f_{sk} / \gamma_s$ $= [6971 * 250 / 1.15 + 0.85 * 114913 * 25 / 1.5 + 616 * 415 / 1.15] / 1000$ $= 3366 \text{ kN}$			$P_p = 3366 \text{ kN}$
<p><b>(2) Effective elastic flexural stiffness of the section for short term loading</b></p> <p><u>About the major axis</u></p> $(EI)_{ex} = E_a I_{ax} + 0.8 E_{cd} I_{cx} + E_s I_{sx}$ $I_{ax} = 79.8 * 10^6 \text{ mm}^4$			$E_{cd}$ $= E_{cm} / \gamma_c^*$ $= 31220 / 1.35$ $= 23125 \text{ N/mm}^2$

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$I_{sx} = Ah^2$ $= 616 * [ 350/2-25-7]^2$ $= 12.6 * 10^6 mm^4$			
$I_{cx} = (350)^4/12 - [ 79.8 + 12.6] * 10^6$ $= 1158 * 10^6 mm^4$			
$(EI)_{ex} = 2.0 * 10^5 * 79.8 * 10^6 + 0.8 * 23125 * 1158 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$ $= 39.4 * 10^{12} N mm^2$			
<p><u>About minor axis</u></p>			
$(EI)_{ey} = 2.0 * 10^5 * 20.1 * 10^6 + 0.8 * 23125 * 1217.8 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$ $= 28.5 * 10^{12} N mm^2$			
<p><b>(3) Non dimensional slenderness</b></p>			
$\bar{\lambda} = (P_{pu} / P_{cr})^{1/2}$			
<p>Value of <math>P_{pu}</math>:</p>			
$P_{pu} = A_d f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$			
$P_{pu} = A_d f_y + \alpha_c A_c * 0.80 * (f_{ck})_{cu} + A_s f_{sk}$			
$= (6971 * 250 + 0.85 * 114913 * 25 + 415 * 616) / 1000$			
$= 4440 kN$			$P_{pu} = 4440 kN$
$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$			
$= \frac{\pi^2 * 39.4 * 10^{12}}{(3000)^2} = 43207 kN$			$(P_{cr})_x = 43207 kN$

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$(P_{cr})_y = \frac{\pi^2 * 28.5 * 10^{12}}{(3000)^2} = 31254 \text{ kN}$		$(P_{cr})_y = 31254 \text{ kN}$	
$\bar{\lambda}_x = (44.4 / 432.07)^{1/2} = 0.320$		$\bar{\lambda}_x = 0.320$	
$\bar{\lambda}_y = (44.4 / 312.54)^{1/2} = 0.377$		$\bar{\lambda}_y = 0.377$	
<p><b>(4) Check for the effect of long term loading</b></p> <p><i>The effect of long term loading can be neglected if anyone or both following conditions are satisfied:</i></p> <ul style="list-style-type: none"> <li><i>Eccentricity, e given by</i> <math display="block">e = M / P \geq 2 \text{ times the cross section dimension in the plane of bending considered.}</math> <math display="block">e_x = 180/1500 = 0.12 &lt; 2(0.35)</math> <math display="block">e_y = 0</math> </li> <li><math>\bar{\lambda} &lt; 0.8</math> <p><i>Since condition (2) is satisfied, the influence of creep and shrinkage on the ultimate load need not be considered.</i></p> </li> </ul>			
<p><b>(5) Resistance of the composite column under axial compression</b></p> <p><i>Design against axial compression is satisfied if following condition is satisfied:</i></p> $P < \chi P_p$			

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<p><i>Here,</i></p> <p><math>P = 1500 \text{ kN}</math></p> <p><math>P_p = 3366 \text{ kN}</math></p> <p><i>and <math>\chi</math> = reduction factor for column buckling</i></p> <p><i><math>\chi</math> values:</i></p> <p><u>About major axis</u></p> <p><math>\alpha_x = 0.34</math></p> $\chi_x = 1 / \{ \phi_x + (\phi_x^2 - \bar{\lambda}_x^2)^{1/2} \}$ $\phi_x = 0.5 [1 + \alpha_x (\bar{\lambda}_x - 0.2) + \bar{\lambda}_x^2]$ $= 0.5 [1 + 0.34(0.320 - 0.2) + (0.320)^2] = 0.572$ $\chi_x = 1 / \{ 0.572 + [(0.572)^2 - (0.326)^2]^{1/2} \}$ $= 0.956$ <p><math>\chi_x P_p &gt; P</math></p> $0.956 * 3366 = 3218 \text{ kN} > P (=1500 \text{ kN})$ <p><u>About minor axis</u></p> <p><math>\alpha_y = 0.49</math></p> $\phi_y = 0.5 [1 + 0.49(0.377 - 0.2) + (0.377)^2]$ $= 0.61$ $\chi_y = 1 / \{ 0.61 + [(0.61)^2 - (0.377)^2]^{1/2} \}$ $= 0.918$			



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Checked By		<i>RN</i>	Date
<p><math>\chi_y P_P &gt; P</math></p> <p><math>0.918 * 3366 = 3090 \text{ kN} &gt; P (=1500 \text{ kN})</math></p> <p><math>\therefore</math> The design is OK for axial compression.</p> <p><b>(6) Check for second order effects</b></p> <p><i>Isolated non – sway columns need not be checked for second order effects if:</i></p> <p><math>P / P_{cr} \leq 0.1</math> for major axis bending</p> <p><math>1500/43207 = 0.035 &lt; 0.1</math></p> <p><math>\therefore</math> Check for second order effects is not necessary</p> <p><b>(7) Resistance of the composite column under axial compression and uni-axial bending</b></p> <p><i>Compressive resistance of concrete, <math>P_c = A_c p_{ck}</math></i>  <math>= 1628 \text{ kN}</math></p> <p><i>Plastic section modulus of the reinforcement</i>  <math>Z_{ps} = 4(\pi / 4 * 14^2) * (350/2 - 25 - 14/2)</math>  <math>= 88 * 10^3 \text{ mm}^3</math></p> <p><i>Plastic section modulus of the steel section</i>  <math>Z_{pa} = 699.8 * 10^3 \text{ mm}^3</math></p> <p><i>Plastic section modulus of the concrete</i>  <math>Z_{pc} = bch_c^2 / 4 - Z_{ps} - Z_{pa}</math>  <math>= (350)^3 / 4 - 88 * 10^3 - 699.8 * 10^3</math>  <math>= 9931 * 10^3 \text{ mm}^3</math></p>			

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<p><i>Check that the position of neutral axis is in the web</i></p> $h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w (2p_y - p_{ck})}$ $= \frac{114913 * \frac{0.85 * 25}{1.5}}{2 * 350 * \frac{0.85 * 25}{1.5} + 2 * 8.8 ( 2 * \frac{250}{1.15} - \frac{0.85 * 25}{1.5} )}$ $= 93.99 \text{ mm} < (h/2 - t_f) = \left( \frac{250}{2} - 9.7 \right) = 115.3 \text{ mm}$ <p><i>The neutral axis is in the web.</i></p> <p><i>A'_s = 0 as there is no reinforcement with in the region of the steel web</i></p> <p><u>Section modulus about neutral axis</u></p> <p><i>Z<sub>psn</sub> = 0 (As there is no reinforcement with in the region of 2h<sub>n</sub> from the middle line of the cross section)</i></p> $Z_{pan} = t_w h_n^2 = 8.8 * (93.99)^2$ $= 77740.3 \text{ mm}^3$ $Z_{pcn} = b_c h_n^2 - Z_{psn} - Z_{pan}$ $= 350 (93.99)^2 - 77740. = 3014.2 * 10^3 \text{ mm}^3$ <p><i>Plastic moment resistance of section</i></p> $M_p = p_y ( Z_{pa} - Z_{pan} ) + 0.5 p_{ck} ( Z_{pc} - Z_{pcn} ) + p_{sk} ( Z_{ps} - Z_{psn} )$ $= 217.4 (699800 - 77740) + 0.5 * 0.85 * 25 / 1.5 (9931000 - 3014200)$ $+ 361 (88 * 1000)$ $= 216 \text{ kNm}$			

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<p><b>(8) Check of column resistance against combined compression and uni-axial bending</b></p> <p><i>The design against combined compression and uni-axial bending is adequate if following condition is satisfied:</i></p> $M \leq 0.9 \mu M_p$ <p><math>M = 180 \text{ kNm}</math>  <math>M_p = 216 \text{ kNm}</math>  <math>\mu = \text{moment resistance ratio}</math>  <math>= 1 - \frac{(1 - \chi) \chi_d}{(1 - \chi_c) \chi}</math>  <math>= 1 - \frac{(1 - 0.956) 0.446}{(1 - 0.484) 0.956}</math>  <math>= 0.960</math></p> <p><math>\therefore M &lt; 0.9 \mu M_p</math>  <math>&lt; 0.9 (0.960) * (216)</math>  <math>&lt; 187 \text{ kNm}</math></p> <p><i>Hence the composite column is acceptable and the check is satisfied.</i></p>			
		$\chi_d = P / P_p$ $= 1500 / 3366$ $= 0.446$ $\chi_c = P_c / P_p$ $= 1628 / 3366$ $= 0.484$	

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<h2>Calculation Sheet</h2>	Checked By <i>RN</i>		Date																							
	<p><u><b>PROBLEM 2</b></u></p> <p><i>Check the adequacy of the concrete encased composite section shown below for bi-axial bending</i></p> <div style="text-align: center;"> <p>ISHB 250                      4 of 14 <math>\phi</math> bars</p> </div> <p><b>4.2.1 DETAILS OF THE SECTION</b></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 30%;"><i>Column dimension</i></td> <td style="width: 40%;">350 X 350 X 3000</td> <td style="width: 30%;"></td> </tr> <tr> <td><i>Concrete Grade</i></td> <td>M30</td> <td></td> </tr> <tr> <td><i>Steel Section</i></td> <td>ISHB 250</td> <td></td> </tr> <tr> <td><i>Steel Reinforcement</i></td> <td>Fe415</td> <td></td> </tr> <tr> <td></td> <td>4 Nos. of 14 mm dia bar</td> <td></td> </tr> <tr> <td><i>Design Axial Load</i></td> <td>1500 kN</td> <td rowspan="4" style="vertical-align: middle;"> <i>Axial Load</i>                      = 1500kN  <math>M_x</math>                      = 180kNm   <math>M_y = 120kNm</math> </td> </tr> <tr> <td><i>Design bending moment about x-x axis</i></td> <td>180 kNm</td> </tr> <tr> <td><i>Design bending moment about y-y axis</i></td> <td>120 kNm</td> </tr> <tr> <td></td> <td></td> </tr> </table>			<i>Column dimension</i>	350 X 350 X 3000		<i>Concrete Grade</i>	M30		<i>Steel Section</i>	ISHB 250		<i>Steel Reinforcement</i>	Fe415			4 Nos. of 14 mm dia bar		<i>Design Axial Load</i>	1500 kN	<i>Axial Load</i> = 1500kN $M_x$ = 180kNm  $M_y = 120kNm$	<i>Design bending moment about x-x axis</i>	180 kNm	<i>Design bending moment about y-y axis</i>	120 kNm	
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<p><b><u>DESIGN CALCULATIONS:</u></b></p> <p><b>4.2.2 LIST MATERIAL PROPERTIES</b></p> <p><b>(1) Structural steel</b></p> <p><i>Steel section ISHB 250</i>  <i>Nominal yield strength <math>f_y = 250 \text{ N/mm}^2</math></i>  <i>Modulus of elasticity <math>E_a = 200 \text{ kN/mm}^2</math></i></p> <p><b>Concrete</b></p> <p><i>Concrete grade M30</i>  <i>Characteristic strength <math>(f_{ck})_{cu} = 30 \text{ N/mm}^2</math></i>  <i>Secant modulus of elasticity for short term loading, <math>E_{cm} = 31220 \text{ N/mm}^2</math></i></p> <p><b>Reinforcing steel</b></p> <p><i>Steel grade Fe 415</i>  <i>Characteristic strength <math>f_{sk} = 415 \text{ N/mm}^2</math></i>  <i>Modulus of elasticity <math>E_s = 200 \text{ kN/mm}^2</math></i></p> <p><b>Partial safety factors</b></p> <p><math>\gamma_a = 1.15</math>  <math>\gamma_c = 1.5</math>  <math>\gamma_s = 1.15</math></p> <p><b>4.2.3 LIST SECTION PROPERTIES OF THE GIVEN SECTION</b></p> <p><b>(1) Steel section</b></p> <p><math>A_a = 6971 \text{ mm}^2</math></p>			

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<h2>Calculation Sheet</h2>		Checked By <i>RN</i>	Date
<p> <math>t_f = 9.7 \text{ mm}</math>  <math>h = 250 \text{ mm}</math>  <math>t_w = 8.8 \text{ mm}</math>  <math>I_{ax} = 79.8 * 10^6 \text{ mm}^4</math>  <math>I_{ay} = 20.1 * 10^6 \text{ mm}^4</math>  <math>Z_{px} = 699.8 * 10^3 \text{ mm}^3</math>  <math>Z_{py} = 307.6 * 10^3 \text{ mm}^3</math> </p> <p><b>(2) Reinforcing steel</b></p> <p>4 bars of 14 mm dia, <math>A_s = 616 \text{ mm}^2</math></p> <p><b>(3) Concrete</b></p> $A_c = A_{gross} - A_a - A_s$ $= 350 * 350 - 6971 - 616$ $= 114913 \text{ mm}^2$ <p><b>4.2.4 DESIGN CHECKS</b></p> <p><b>(1) Plastic resistance of the section</b></p> $P_p = A_a f_y / \gamma_a + \alpha_c A_c (f_{ck})_{cy} / \gamma_c + A_s f_{sk} / \gamma_s$ $P_p = A_a f_y / \gamma_a + \alpha_c A_c 0.80 * (f_{ck})_{cu} / \gamma_c + A_s f_{sk} / \gamma_s$ $= [6971 * 250 / 1.15 + 0.85 * 114913 * 25 / 1.5 + 616 * 415 / 1.15] / 1000$ $= 3366 \text{ kN}$ <p><b>(3) Effective elastic flexural stiffness of the section for short term loading</b></p> <p><u>About the major axis</u></p> $(EI)_{ex} = E_a I_{ax} + 0.8 E_{cd} I_{cx} + E_s I_{sx}$ $I_{ax} = 79.8 * 10^6 \text{ mm}^4$			<p><math>P_p = 3366 \text{ kN}</math></p> <p><math>E_{cd} = E_{cm} / \gamma_c^*</math></p> $= 31220 / 1.35$ $= 23125 \text{ N/mm}^2$

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		Made By <i>PU</i>	Date
<h2>Calculation Sheet</h2>		Checked By <i>RN</i>	Date
$I_{xx} = Ah^2$ $= 616 * [ 350/2-25-7]^2$ $= 12.6 * 10^6 mm^4$			
$I_{cx} = (350)^4/12 - [ 79.8 + 12.6] * 10^6$ $= 1158 * 10^6 mm^4$			
$(EI)_{ex} = 2.0 * 10^5 * 79.8 * 10^6 + 0.8 * 23125 * 1158 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$ $= 39.4 * 10^{12} N mm^2$			
<p><u>About minor axis</u></p>			
$(EI)_{ey} = 2.0 * 10^5 * 20.1 * 10^6 + 0.8 * 23125 * 1217.8 * 10^6 + 2.0 * 10^5 * 12.6 * 10^6$ $= 28.5 * 10^{12} N mm^2$			
<p><b>(4) Non dimensional slenderness</b></p>			
$\bar{\lambda} = (P_{pu}/P_{cr})^{1/2}$			
<p>Value of <math>P_{pu}</math>:</p>			
$P_{pu} = A_a f_y + \alpha_c A_c (f_{ck})_{cy} + A_s f_{sk}$			
$P_{pu} = A_a f_y + \alpha_c A_c 0.80 * (f_{ck})_{cu} + A_s f_{sk}$			
$= (6971 * 250 + 0.85 * 114913 * 25 + 415 * 616)/1000$			
$= 4440 kN$			$P_{pu} = 4440 kN$
$(P_{cr})_x = \frac{\pi^2 (EI)_{ex}}{\ell^2}$			
$= \frac{\pi^2 * 39.4 * 10^{12}}{(3000)^2}$			
$= 43207 kN$			$(P_{cr})_x = 43207 kN$

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	Worked Example <b>2</b>		
		Made By <i>PU</i>	Date
<h2>Calculation Sheet</h2>		Checked By <i>RN</i>	Date
$(P_{cr})_y = \frac{\pi^2 * 28.5 * 10^{12}}{(3000)^2} = 31254kN$		$(P_{cr})_y = 31254 kN$	
$\bar{\lambda}_x = (44.4 / 432.07)^{1/2} = 0.320$		$\bar{\lambda}_x = 0.320$	
$\bar{\lambda}_y = (44.4/312.54)^{1/2} = 0.377$		$\bar{\lambda}_y = 0.377$	
<p><b>(5) Check for the effect of long term loading</b></p> <p><i>The effect of long term loading can be neglected if anyone or both following conditions are satisfied:</i></p> <ul style="list-style-type: none"> <li><i>Eccentricity, e given by</i>  <math>e = M / P \geq 2</math> times the cross section dimension in the plane of bending considered)  <math>e_x = 180 / 1500 = 0.12 &lt; 2(0.350)</math>  <math>e_y = 120 / 1500 = 0.08 &lt; 2(0.350)</math></li> <li><math>\bar{\lambda} &lt; 0.8</math></li> </ul> <p><i>Since condition (2) is satisfied, the influence of creep and shrinkage on the ultimate load need not be considered.</i></p>			
<p><b>(6) Resistance of the composite column under axial compression</b></p> <p><i>Design against axial compression is satisfied if following condition is satisfied:</i></p> $P < \chi P_p$			



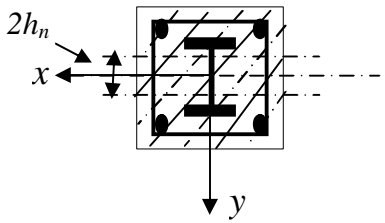
<b>Structural Steel Design Project</b>	Job No:	Sheet <i>6 of 11</i>	Rev
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	Worked Example <b>2</b>		
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<b>Calculation Sheet</b>		Checked By <i>RN</i>	Date
<p><i>Here,</i> <math>P = 1500 \text{ kN}</math></p> <p><math>P_p = 3366 \text{ kN}</math></p> <p><i>and <math>\chi</math> = reduction factor for column buckling</i></p> <p><i><math>\chi</math> values:</i></p> <p><u>About major axis</u></p> <p><math>\alpha_x = 0.34</math>  <math>\chi_x = 1 / \{ \phi + (\phi^2 - \bar{\lambda}_x^2)^{1/2} \}</math></p> <p><math>\phi_x = 0.5 [1 + \alpha_x (\bar{\lambda}_x - 0.2) + \bar{\lambda}_x^2]</math>  <math>= 0.5 [1 + 0.34(0.320 - 0.2) + (0.320)^2] = 0.572</math></p> <p><math>\chi_x = 1 / \{ 0.572 + [(0.572)^2 - (0.320)^2]^{1/2} \}</math>  <math>= 0.956</math></p> <p><math>\chi_x P_{Px} &gt; P</math></p> <p><math>0.956 * 3366 = 3218 \text{ kN} &gt; P (=1500 \text{ kN})</math></p> <p><u>About minor axis</u></p> <p><math>\alpha_y = 0.49</math></p> <p><math>\phi_y = 0.5 [1 + 0.49(0.377 - 0.2) + (0.377)^2]</math>  <math>= 0.61</math></p> <p><math>\chi_y = 1 / \{ 0.61 + [(0.61)^2 - (0.377)^2]^{1/2} \}</math>  <math>= 0.918</math></p>			

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		Made By <i>PU</i>	Date
<h2>Calculation Sheet</h2>		Checked By <i>RN</i>	Date
<p><math>\chi_y P_{py} &gt; P</math></p> <p><math>0.918 * 3366 = 3090 \text{ kN} &gt; P (=1500 \text{ kN})</math>  <math>\therefore</math> The design is OK for axial compression.</p> <p><b>(7) Check for second order effects</b></p> <p><i>Isolated non – sway columns need not be checked for second order effects if:</i></p> <p><math>P/(P_{cr})_x \leq 0.1</math> for major axis bending  <math>1500 / 43207 = 0.035 \leq 0.1</math></p> <p><math>P/(P_{cr})_y \leq 0.1</math> for minor axis bending  <math>1500 / 31254 = 0.048 \leq 0.1</math></p> <p><math>\therefore</math> Check for second order effects is not necessary</p> <p><b>(8) Resistance of the composite column under axial compression and bi-axial bending</b></p> <p>Compressive resistance of concrete, <math>P_c = A_c p_{ck}</math>  <math>= 1628 \text{ kN}</math></p> <p><u>About Major axis</u></p> <p>Plastic section modulus of the reinforcement  <math>Z_{ps} = 4(\pi / 4 * 14^2) * (350/2 - 25 - 14/2)</math>  <math>= 88 * 10^3 \text{ mm}^3</math></p> <p>Plastic section modulus of the steel section  <math>Z_{pa} = 699.8 * 10^3 \text{ mm}^3</math></p> <p>Plastic section modulus of the concrete  <math>Z_{pc} = b_c h_c^2 / 4 - Z_{ps} - Z_{pa}</math>  <math>= (350)^3 / 4 - 88 * 10^3 - 699.765 * 10^3</math>  <math>= 9931 * 10^3 \text{ mm}^3</math></p>			

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<h2>Calculation Sheet</h2>		Checked By <i>RN</i>	Date

*Check that the position of neutral axis is in the web*



$$h_n = \frac{A_c p_{ck} - A'_s (2p_{sk} - p_{ck})}{2b_c p_{ck} + 2t_w (2p_y - p_{ck})}$$

$$= \frac{114913 * \frac{0.85 * 25}{1.5}}{2 * 350 * \frac{0.85 * 25}{1.5} + 2 * 8.8 ( 2 * \frac{250}{1.15} - \frac{0.85 * 25}{1.5} )}$$

$$= 93.99 \text{ mm} < (h/2 - t_f) = \left( \frac{250}{2} - 9.7 \right) = 115.3 \text{ mm}$$

*The neutral axis is in the web*

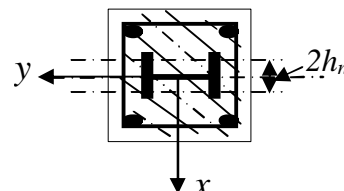
$A'_s = 0$  as there is no reinforcement with in the region of the steel web

Section modulus about neutral axis

$Z_{psn} = 0$  (As there is no reinforcement with in the region of  $2h_n$  from the middle line of the cross section)

$$Z_{pan} = t_w h_n^2 = 8.8 * (93.99)^2 = 77740.3 \text{ mm}^3$$

$$Z_{pcn} = b_c h_n^2 - Z_{psn} - Z_{pan} = 350 (93.99)^2 - 77740.3 = 3014.2 * 10^3 \text{ mm}^3$$

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		Made By <i>PU</i>	Date
<h2>Calculation Sheet</h2>		Checked By <i>RN</i>	Date
<p><i>Plastic moment resistance of section</i></p> $M_p = p_y ( Z_{pa} - Z_{pan} ) + 0.5 p_{ck} ( Z_{pc} - Z_{pcn} ) + p_{sk} ( Z_{ps} - Z_{psn} )$ $= 217.4 ( 699800 - 77740.3 ) + 0.5 * 0.85 * 25 / 1.5 ( 9931000 - 3014200 ) + 361 ( 88 * 1000 )$ $= 216 \text{ kNm}$ <p><u><i>About minor axis</i></u></p> <p><i>Plastic section modulus of the reinforcement</i></p> $Z_{ps} = 4 ( \pi / 4 * 14^2 ) * ( 350 / 2 - 25 - 14 / 2 )$ $= 88 * 10^3 \text{ mm}^3$ <p><i>Plastic section modulus of the steel section</i></p> $Z_{pa} = 307.6 * 10^3 \text{ mm}^3$ <p><i>Plastic section modulus of the concrete</i></p> $Z_{pc} = b_c h_c^2 / 4 - Z_{ps} - Z_{pa}$ $= ( 350 )^3 / 4 - 88 * 10^3 - 307.6 * 10^3$ $= 10323 * 10^3 \text{ mm}^3$ <div style="text-align: center;">  </div> $h_n = \frac{ A_c p_{ck} - A'_s ( 2 p_{sk} - p_{ck} ) + t_w ( 2 t_f - h ) ( 2 p_y - p_{ck} ) }{ 2 h_c p_{ck} + 4 t_f ( 2 p_y - p_{ck} ) }$			

<h1>Structural Steel Design Project</h1>	Job No:	Sheet <i>10 of 11</i>	Rev
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	Worked Example		
		Made By <i>PU</i>	Date
<h2>Calculation Sheet</h2>		Checked By <i>RN</i>	Date
$h_n = \frac{114913 * 14.2 + 8.8(2 * 9.7 - 250)(2 * 218 - 14.2)}{2 * 350 * 14.2 + 4 * 9.7 ( 2 * 218 - 14.2 )}$ $= 29.5 \text{ mm} \left( t_w / 2 < h_n < b / 2 \right) = 8.8 / 2 < h_n < 250 / 2$ <p><math>A'_s = 0</math> as there is no reinforcement with in the region of the steel web</p> <p style="text-align: center;"><u>Section modulus about neutral axis</u></p> <p><math>Z_{psn} = 0</math> (As there is no reinforcement with in the region of <math>2h_n</math> from the middle line of the cross section)</p> $Z_{pan} = 2t_f h_n^2 + (h - 2t_f) / 4 * t_w^2$ $= 2(9.7)(29.5)^2 + [(250 - 2(9.7)) / 4] * 8.8^2$ $= 21.3 * 10^3 \text{ mm}^3$ $Z_{pcn} = h_c h_n^2 - Z_{psn} - Z_{pan}$ $= 350 (29.5)^2 - 21.3 * 10^3$ $= 283.3 * 10^3 \text{ mm}^3$ $M_{py} = p_y ( Z_{pa} - Z_{pan} ) + 0.5 p_{ck} ( Z_{pc} - Z_{pcn} ) + p_{sk} ( Z_{ps} - Z_{psn} )$ $= 217.4 (307.589 - 21.3) * 10^3 + 0.5 * 14.2 * (10323 - 283.3) * 10^3 + 361 (88 * 1000)$ $= 165 \text{ kNm}$ <p><b>(9) Check of column resistance against combined compression and bi-axial bending</b></p> <p>The design against combined compression and bi-axial bending is adequate if following conditions are satisfied:</p> <p>(1) <math>M \leq 0.9 \mu M_p</math></p> <p style="text-align: center;"><u>About major axis</u></p> $M_x = 180 \text{ kNm}$			

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<h2>Calculation Sheet</h2>		Checked By <i>RN</i>	Date
$M_{px} = 216 \text{ kNm}$  $\mu_x = \text{moment resistance ratio}$ $= 1 - \frac{(1 - \chi_x) \chi_d}{(1 - \chi_c) \chi_x}$ $= 1 - \frac{(1 - 0.956) 0.446}{(1 - 0.484) 0.956}$ $= 0.960$  $\therefore M_x < 0.9 \mu_x M_{px}$  $< 0.9 (0.960) * (216)$ $= 187 \text{ kNm}$  <u>About minor axis</u>  $M_y = 120 \text{ kNm}$ $M_{py} = 165 \text{ kNm}$  $\mu_y = 1 - \frac{(1 - \chi_y) \chi_d}{(1 - \chi_c) \chi_y}$ $= 1 - \frac{(1 - 0.918) 0.446}{(1 - 0.448) 0.918}$ $= 0.928$ $\therefore M_y < 0.9 \mu_y M_{py}$  $< 0.9 (0.928) * (165)$ $< 138 \text{ kNm } (M_y = 120 \text{ kN})$  $(2) \frac{M_x}{\mu_x M_{px}} + \frac{M_y}{\mu_y M_{py}} \leq 1.0$  $\frac{180}{0.960 * 216} + \frac{120}{0.928 * 165} > 1.0$		$\chi_d = P/P_p$ $= 1500/3366$ $= 0.446$  $\chi_c = P_c/P_p$ $= 1628/3366$ $= 0.484$	
<p>Since design check (2) is not satisfied, the composite column is not acceptable.</p>			