40 MULTI – STOREY BUILDINGS – IV

1.0 INTRODUCTION

Historically monuments such as the pyramids of Egypt, Taj Mahal in India, the Temples of Greece, the Viaduct of Rome were all built principally with masonry using some form of stone or moulded bricks. Since the introduction of Besemer's process in 1856 several tall buildings have been built using steel. The *381 m* Empire State Building, the Twin Towers of the World Trade Centre and the Sears Tower in Chicago completed in 1974 have clearly established the suitability of steel frame construction for Tall Buildings.

The innovations in lateral load resisting systems (such as introduction of frame-wall, framed tube, belt truss with outrigger, tube in tube and bundled tube systems) to cater to different storey heights and environmental requirements based on susceptibility of structures to either wind or seismic effects, have made it possible to build tallest buildings in the world using steel frames. The advancements in computer techniques and the interaction of Computer Aided Design and Computer Aided Manufacture are likely to have their impact on improved fabrication and erection techniques in reaching even taller structures using steel frames in the foreseeable future.

When we build such tall structures it becomes necessary to consider some of the effects such as the effect of lateral deflection, Δ on gravity loading, P which are normally ignored in the design of building frames of normal height (say three or four storeys).

A building frame deflects under lateral load. The columns of tall buildings carry large axial loads. A building frame, which deflects under lateral load, is further forced to undergo additional deflection because of the eccentricity of gravity load from the centre of gravity of the column due to the deflected shape. These two effects of large axial loads P in the columns combined with significant lateral deflection Δ needs careful consideration in the design of tall multi-storey buildings. The combined effect of the large axial loads P and lateral deflection Δ give rise to the destabilising P-Δ effect. However, in frames that are only a few storeys high, this effect is negligible and hence ignored in the analysis. It is therefore necessary to classify frames based on the relative importance P-Δ effects for the purpose of evaluating design forces.

2.0 CLASSIFICATION OF FRAMES

A frame in which sway Δ is prevented is called a "non-sway" frame. However, there are some frames, which may sway only by a small amount since the magnitude of sway in such frame is small it will have only a negligible P-Δ effect. Such frames are also classified as "non-sway" frames. Therefore, to define the non-sway frame precisely, its lateral stiffness is used as the criteria irrespective of whether it is braced or not. For such frames lateral stiffness is provided by one of the following:

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(i) rigidity of the joints.
(ii) provision of bracing system.
(iii) connecting the frame to a braced frame, shear core, shear wall or a lift well.

The inter storey deflection (*s*) (i.e. the difference in deflection of top and bottom end of a column in that storey) is used to quantify the lateral stiffness of the frame. The meaning of inter storey deflection (*s*) is shown in Fig. 1(*c*). Fig.1 (*a*) shows a typical multi-storey frame subjected to factored (dead + live) load. To ascertain the stiffness of the frame, it is analysed when subjected to assumed forces of magnitude *0.5%* of factored (dead + live) load applied laterally on the frame at each floor level as shown in Fig.1 (*b*) for getting the inter storey deflection (s_i) for the ith storey. Note that the lateral loads are applied without the presence of dead and live loads. The maximum s_i for any storey is taken as a measure of the frame stiffness.

For a frame to be of the non-sway type the maximum inter storey deflection permitted in any storey is generally taken as follows:

$$s_i = \frac{h_i}{4000} \text{ for bare frames}$$
$$= \frac{h_i}{2000} \text{ for frames with cladding}$$

where h_i is the height of the ith storey.

3.0 IDEALISATION OF MATERIAL BEHAVIOUR FOR ANALYSIS OF FRAMES

The strength and stability of a rigid jointed frame is examined based on material stress – strain idealisation of its true behaviour.

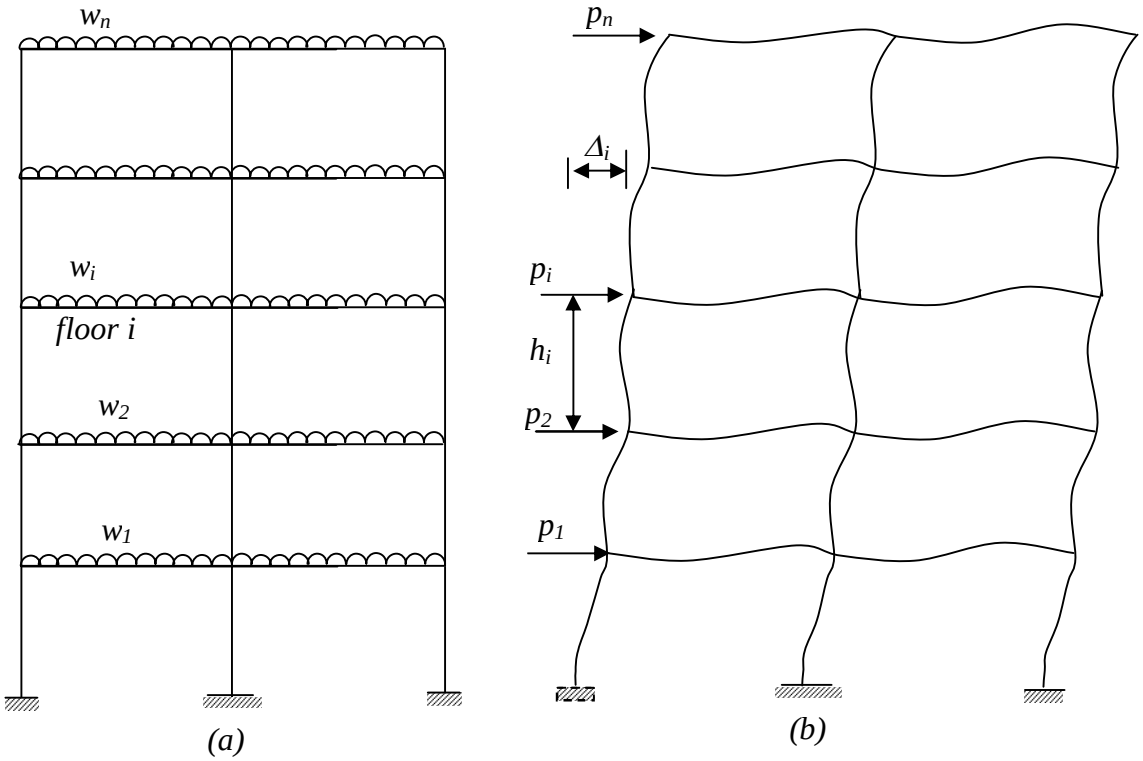
3.1 Elastic Behaviour

Fig.2 (*a*) shows idealised elastic stress–strain behaviour of typical steel for most structures where deformations are not large enough to change the equilibrium equations, this idealised elastic behaviour has been the basis of analysis. This analysis is invalid both in the non-linear range of material behaviour (material non-linearity) or when deformations are so large that the change the line of action of the force requires study of the equilibrium under deformed shape.

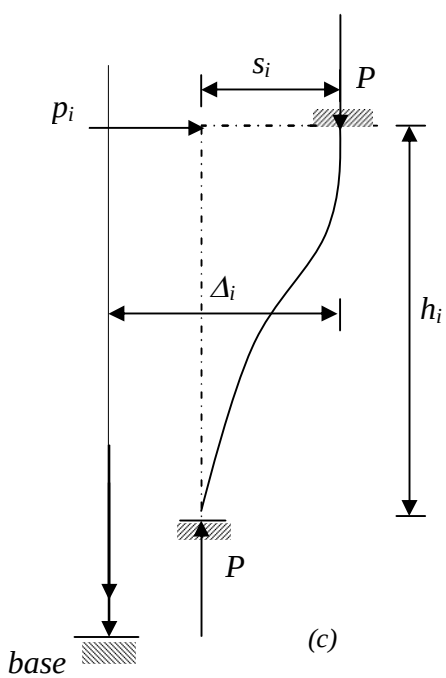
3.2 Elastic – Plastic Behaviour

Fig 2(*b*) shows a more realistic elastic-plastic idealisation. This elastic–plastic idealisation can be used to obtain ultimate load. Within the range of admissible deformations, this idealisation is sufficiently accurate though it does not consider strain-hardening effects. This has been successfully used by researchers for obtaining full range

of behaviour using a realistic mathematical model of the structure. However, it is found to be time consuming and unsuitable for design office work.



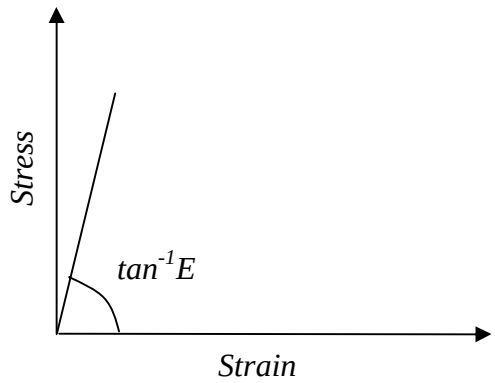
$w_I = \gamma_f (w_{di} + w_{li})$
w_{di} = *dead load on* i^{th} *floor*
w_{li} = *live load on* i^{th} *floor*
w_I = *factored load on* i^{th} *floor*



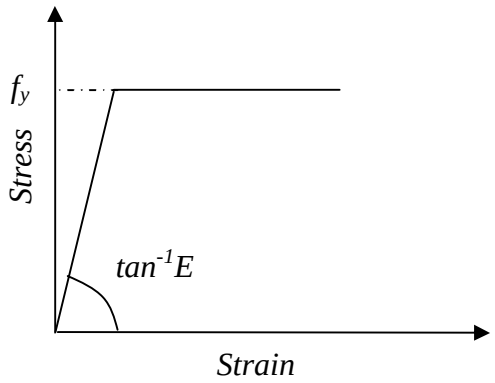
Δ_i = *Total* i^{th} *storey deflection*
S_i = *Interstorey deflection for* i^{th} *floor*
P = *Column Load*
p_i = *Assumed lateral load 0.5% of factored load*

$$= \frac{0.5}{100} * W_i$$

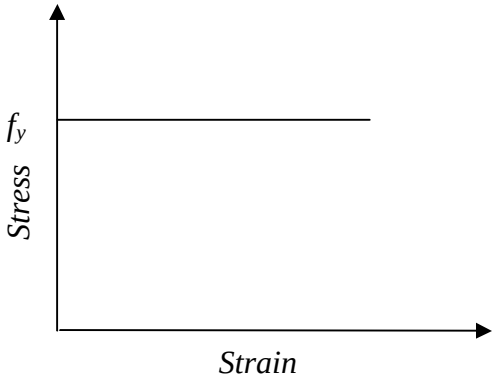
Fig.1 Approximate calculation of frame stiffness for classification of frames



(*a*) *Elastic behaviour*



(*b*) *Elastic-Plastic behaviour*



(*c*) *Rigid-Plastic behaviour*

Fig2: Idealisation of Material Behaviour curve

3.3 Rigid Plastic Behaviour

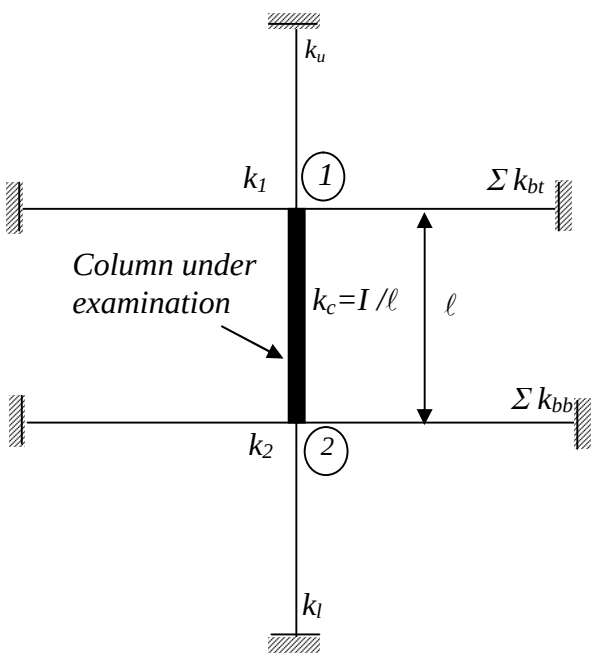
In order to have a realistic estimate of collapse load based on mechanism of failure, rigid-plastic stress-strain idealisation shown in Fig. 2(*c*) has been used. This gives acceptable results for non-sway frames. In tall multi-storey frames, the sway deflections affect the equilibrium equations. Thus mere consideration of rigid-plastic idealisation grossly over estimates the collapse load.

In estimating the realistic collapse load, it has been shown by Horne that the results based on perfectly elastic as well as rigid-plastic idealisations can be combined to give acceptable estimate of actual collapse load for the cases of sway frames. Therefore, it is necessary to study the behaviour of the frame under these idealised material behaviour conditions.

4.0 EFFECTIVE LENGTH OF COLUMNS

4.1 Limited Frame Method

The behaviour of a column under compression is largely controlled by its effective length. A number of idealised end conditions such as pinned, fixed, partially fixed, free and supported on rollers, etc., are used in text books to describe the restraint at the two ends of a column. In multi-storey buildings, columns are continuous and beam members frame into them at floor levels connected rigidly. These columns become a part of either a non-sway or sway frame.



$$k_1 = \frac{k_c + k_u}{k_c + k_u + \sum k_{bt}}$$

$$k_2 = \frac{k_c + k_l}{k_c + k_l + \sum k_{bb}}$$

Fig.3 Limited Substitute Frame

The column, which is a part of the multi-storey non-sway frame, can be idealised to be a part of a limited subframe shown in Fig.3. Let ℓ_e be the effective length of the column, ℓ the actual length between floor beams. The effective length factor for the column is defined as $k = \ell_e/\ell$

In the figure k_u and k_l are relative stiffness I/ℓ values for upper and lower column respectively. Σk_{bt} and Σk_{bb} are the sum of I/ℓ values for beams framing into the column under examination at the top and bottom respectively. The joint restraint coefficient k_n for the column at the top and bottom is obtained from

$$k_n = \frac{Column\ \ stiffness\, of\ columns\ \ meeting\, at\, the\ joint}{Total\ \ stiffness\ of\ \ all\, members\ meeting\, at\ joint\ \ n}$$

In Fig.3, k_1 and k_2 represents the joint stiffness of the column *1-2* at the end *1* and *2* respectively.

4.2 Effective length for Non-sway ($k_3 = \infty$) and sway k_3=0 frames

Based on the work of Wood, the value of relative end restraints k_1 and k_2 can be obtained from Contour Plot reproduced in Fig 4 (*b*) and Fig. 5 (*b*) for the non-sway frame shown in Fig.4 (*a*) and for sway frame shown in Fig.5 (*a*). In the case of non-sway frame, stability criteria considered are rotations that take place at top and bottom end of the column for the elastic critical load using stability functions.

However, in the case of sway frames, [Fig.5 (*a*)] in addition to rotations, the effect of lateral deflection has been considered. Subsequently, it was shown by Wood that the plots in Fig.4(*b*) and Fig.5(*b*) can also be used when the columns at the top and (or) bottom are continuous over stories provided that the joint stiffness at top and bottom are correctly accounted for.

The effective length factor for the column $k = \ell_e/\ell$ for non-sway frames lie in the range of "*0.5* to *1.0*". For sway frames the range increases to "*1* to ∞ " indicating clearly the contribution of lateral sway to instability.

<u>Example *1:*</u>

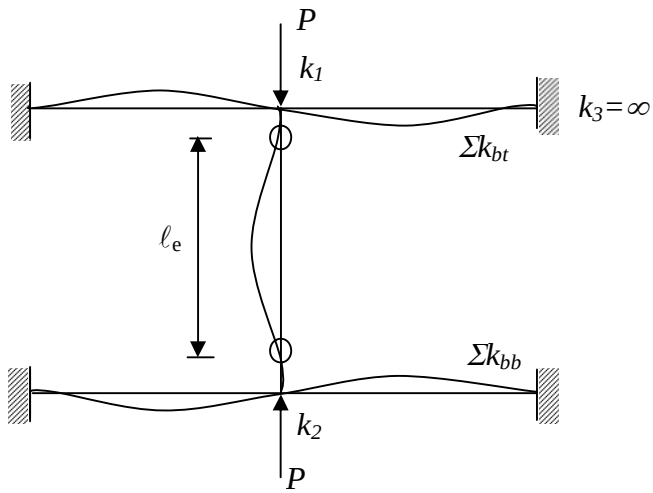
As an example, let us examine the case of a column with $k_1 = \dfrac{k_{co\ell}}{\sum k_{bt} + k_{topcol}} = 0.5$

and $k_2 = \dfrac{k_{co\ell}}{\Sigma k_{bb} + k_{botco\ell}} = 0.6$ for a non-sway frame. The effective length factor k for $k_1 = 0.5$ and $k_2 = 0.6$ and $k_3 = \infty$ (non-sway) from Fig.4(*b*) is *0.72*. Therefore the effective length of the column $\ell_e = 0.72\ \ell$.

For a sway frame ($k_3 = 0$), the effective length factor from Fig. 5(*b*) becomes $k = 1.55$ leading to the effective length of column $\ell_e = 1.55\ \ell$.

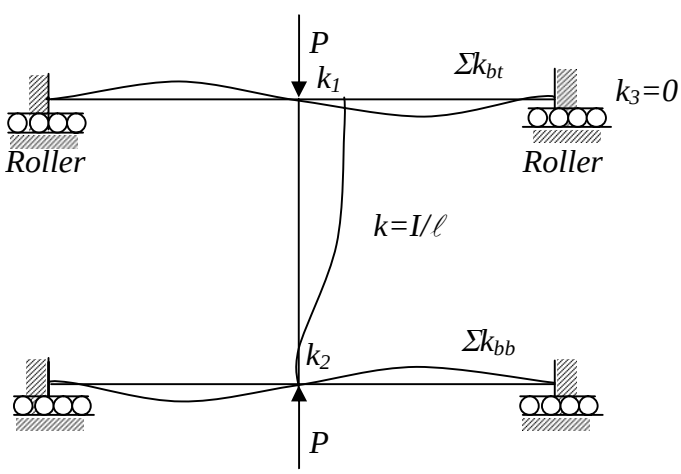
This shows the importance of considering sway for multi-storey columns forming a part of a frame in which $k_2 = \infty$

4.3 Effective length of insufficiently restrained columns in the frames



k_1 =Distribution coefficient at top
Σk_{bt} =Sum of beam stiffness I/ℓ at top
Σk_{bb}=Sum of beam stiffness I/ℓ at bottom
k_2 =Distribution coefficient at bottom
k_3 =Bracing stiffness
$=\infty$ (since braced)

Fig.4(a) Non sway frame ($k_3=\infty$)



k_3 =Bracing stiffness
=0 (since full sway permitted)

Fig.5(a) Sway frame ($k_3=0$)

While using the charts given in Fig.4 (*b*) and Fig.5 (*b*), following limitations should be considered.

(i) When a member is either not present or not firmly connected to the frame, it should be considered to have zero stiffness.

(ii) If a framing member carries nearly full moment (*90%* of its capacity) it will not provide resistance for preventing the column from buckling when plastic hinges have formed. For such beams, stiffness should be taken as zero.

(iii) If the column under question itself carries full moment (*90%* of its capacity) it will develop flexural hinge at top and bottom and as such its effective length should be taken as ℓ.

(iv) When the column is attached to the foundation, a rational value of k at the bottom should be chosen (i.e. $k=1$ if pinned, 0.9 if not rigidly connected and 0.5 if rigidly connected with transverse beams).

The above cases highlight the importance of rotational continuity being distributed by either plasticity or partial release due to practical foundation problems which are likely to reduce the restraint at the ends of the column.

4.4 Effective length consideration when the frame is partially braced

Neither the column considered in Fig.4 (*a*) with full restraint nor the column considered in Fig.5 (*a*) with no restraint can be applied to a case of a frame partially restrained by filler walls in between the framing members. These panel walls partially inhibit sway. In such cases, the effective length will depend on the relative stiffness of bracing system provided.

The relative stiffness of the bracing system to that of the frame is designated as k_3.

BS 5950 gives a method of computing the relative stiffness of the frame based on computed values of the stiffnesses of columns and that of the panels in that storey. This expression given in Equation 1 can be shown to be based on elastic stiffness contribution of the panel to that of the frame:

$$k_3 = \frac{h^2 \Sigma S_p}{80\, E \Sigma k_c} \quad but \leq 2 \qquad (1)$$

where,

h = Storey height

ΣS_p = Sum of the spring stiffness calculated as horizontal force required to produce unit horizontal deflection of the panel in the storey in which the column is located.

E = Modulus of Elasticity of Column

Σk_c = Sum of the stiffness of all columns in that storey represented by their I/ℓ values.

The spring stiffness S_p in equation (1) can be conveniently obtained from the unit load method as given in equation (2)

$$S_p = \frac{0.6\,(h/b)}{\left[1+\left(h/b\right)^2\right]^2}\,t\,E_p \qquad (2)$$

where

h = storey height
b = width of panel
t = thickness of panel
E_p = Modulus of Elasticity of panel

Fig. 6 and Fig.7 show the charts for computing effective length ratios for sway bracing stiffness of $k_3 = 1$ and $k_3 = 2$ respectively. Thus, effective length factor for a column being a part of the frame with $k_3 = 1$ as well as $k_3 = 2$ can be determined using these charts. These charts are intended to account for the effect of partial sway bracing.

The actual effective length factor for the partial sway bracing case for a particular case of bracing stiffness k_3 determined from equation (1) is determined by interpolating the k values obtained for $k_3 = 0$ [Fig 5 (*b*)], $k_3 = 1$ (Fig 6) and $k_3 = 2$ (Fig.7).

Example *2:*

As an example, take the case of $k_1 = 0.5$, $k_2 = 0.6$ for which effective length factor when no bracing is provided was shown to be *1.55*. From Fig.6, for $k_3 = 1$ effective length for $k_1 = 0.5$ and $k_2 = 0.6$ is $1.44\ \ell$. From Fig.7 for $k_3 = 2$ effective length for $k_1 = 0.5$ and $k_2 = 0.6$ is $1.255\ \ell$. If $k_3 = 1.5$ (relative stiffness of bracing to the frame) then the value of effective length factor k will be $\frac{1.44 + 1.255}{2} = 1.3975 \approx 1.4$. Thus the effective length of the column with partial restraint of $k_3 = 1.5$ is $1.4\ \ell$. For this example the effective length factor k for various stiffness of framing system is as shown in Table 1.

Table 1: Effective length factor for the example frame

S.No.	Conditions for lateral restraint	k_1	k_2	k_3	k
1.	Non-sway	*0.5*	*0.6*	∞	*0.72*
2.	Sway	*0.5*	*0.6*	*0*	*1.55*
3.	Partial restraint by panel walls	*0.5*	*0.6*	*1.5*	*1.40*

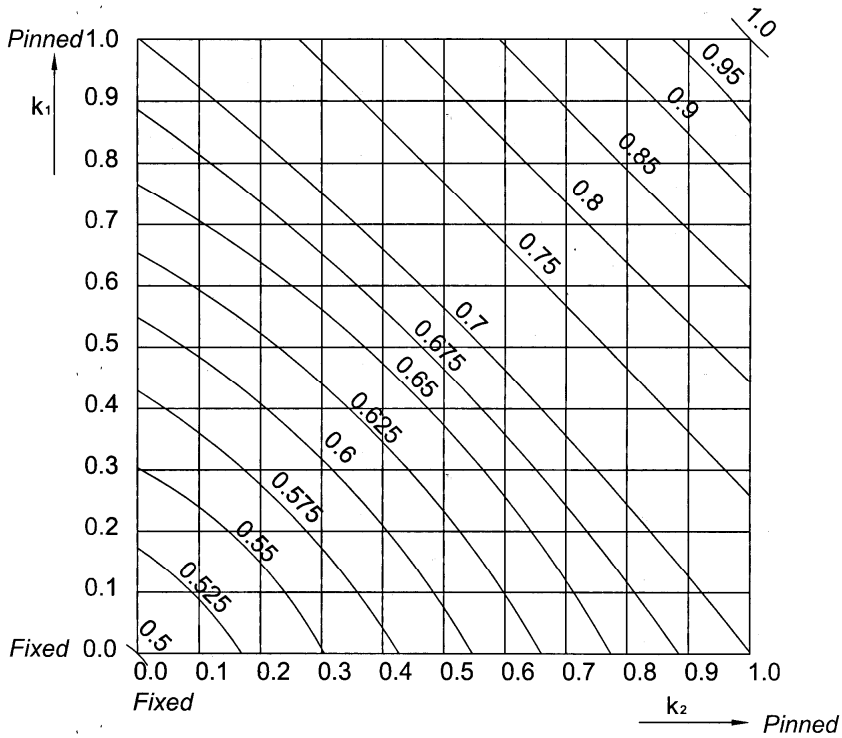


Fig.4 (b) Effective Length ratio ℓ_e/ℓ for a column in a rigid- jointed frame braced against sidesway for $k_3=\infty$

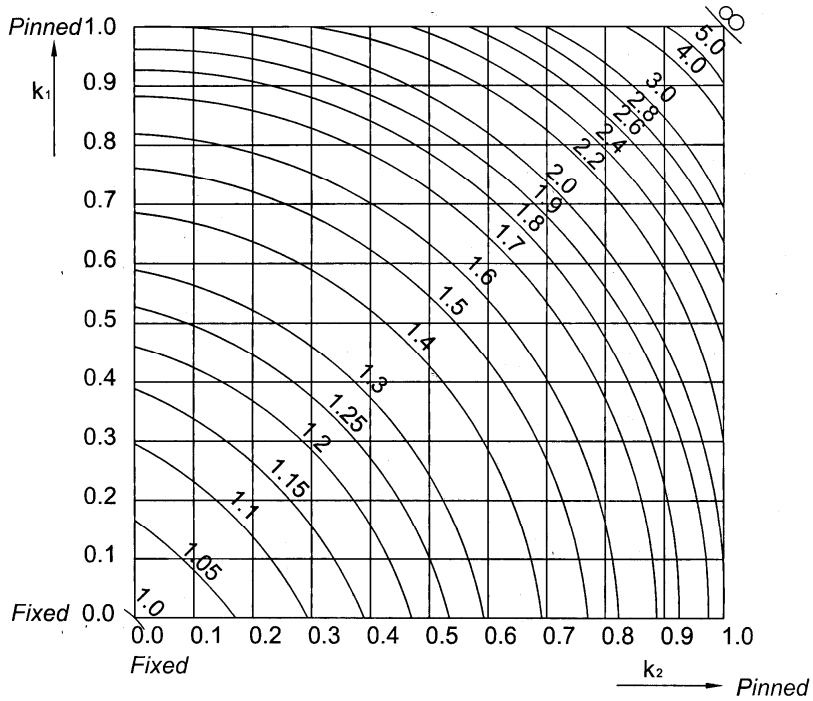


Fig.5 (b) Effective Length ratio ℓ_e/ℓ for a column in a rigid- jointed frame with unrestricted sidesway for $k_3=0$

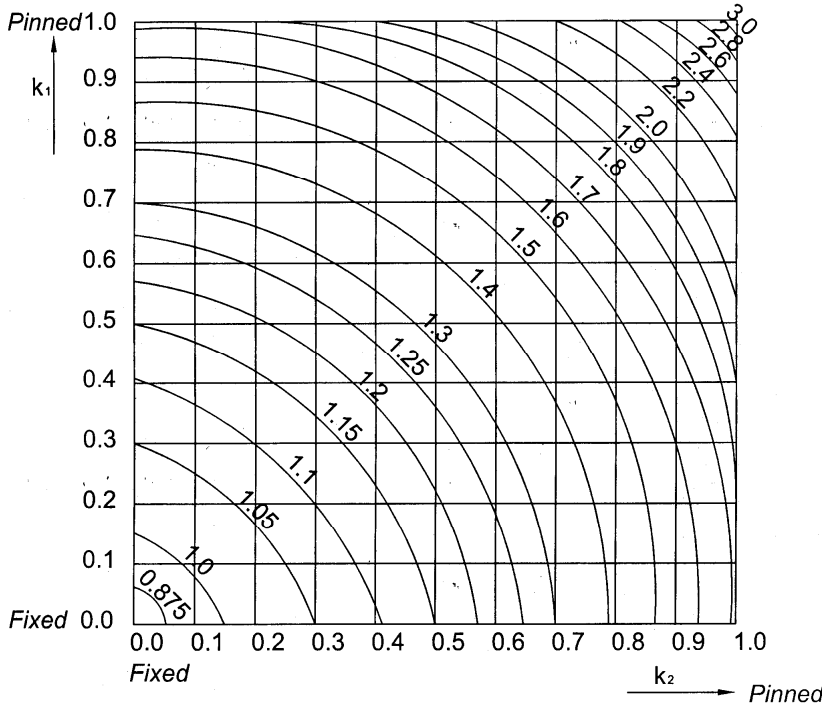


Fig.6 Effective Length ratio ℓ_e/ℓ for a column in a rigid- jointed frame with partial sway bracing of relative stiffness k_3=1

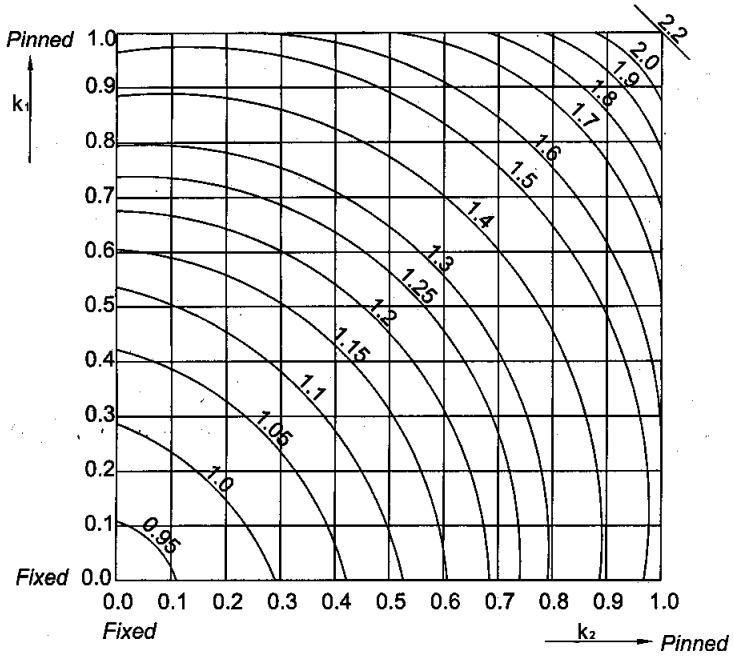


Fig.7 Effective Length ratio ℓ_e/ℓ for a column in a rigid-jointed frame with partial sway bracing of relative stiffness k_3=2

4.5 Consideration of realistic beam stiffness based on buckling mode

It is assumed that the ends of the beam away from the column end under consideration is fully restrained. This assumption is realistic (as shown by Wood) and acceptable because about 48 to 60 percent of the width of slabs are available for stiffening beams and for carrying the fixed end moments of loaded beams. However, this assumption is not appropriate for base frames which are not integral with concrete floor and hence the value I/ℓ used for such floors should be modified taking into account the critical buckling mode at failure.

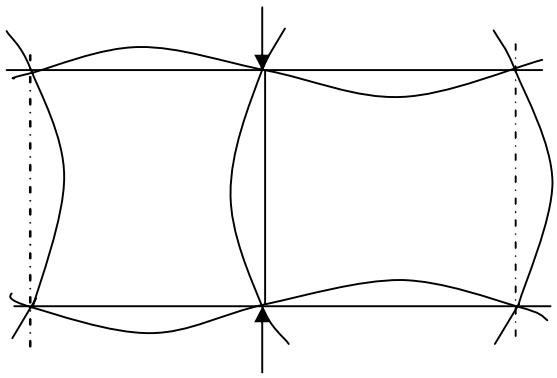


Fig.8 Critical Buckling Mode of a Braced Frame

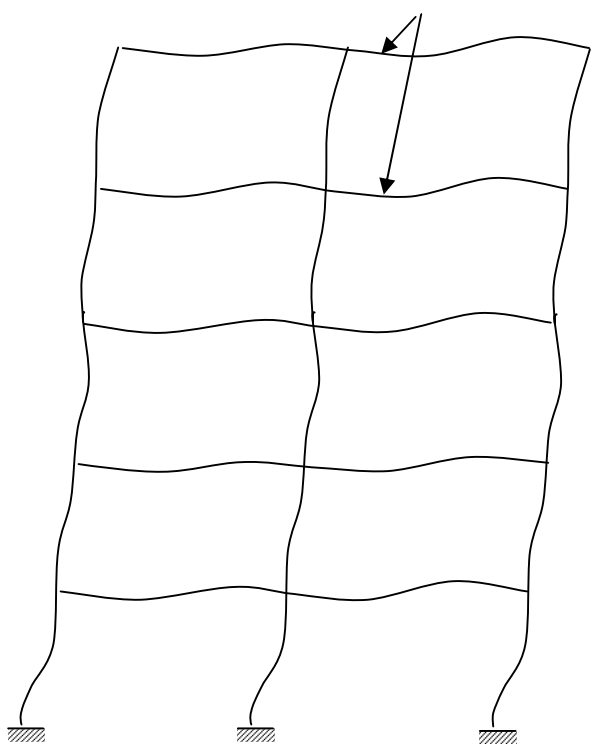


Fig.9 Critical Buckling Mode for an Unbraced Frame

For a non-sway frame, the beams are bent into single curvature as shown in Fig.8. For this case, the beam stiffness is $0.5\ I/\ell$.

In the case of a sway frame, the bending mode will have double curvature as shown in Fig.9. The beam stiffness in this case is $1.5\ I/\ell$. The effective length obtained for the column using this assumption is appropriate. A more exact value can be obtained from the consideration of frame instability discussed later.

It is assumed that the beam members are not subjected to axial forces. In case they are subjected to axial forces, the limited frame method can still be used, provided the frame is a non-sway one and proper care is taken to use reduced stiffness for beams based on the level of axial load carried by it, to its elastic buckling load P_{cr}.

4.6 Method for Determining Effective Length of Columns in Frames according to IS: 800

4.6.1 Method for Determining Effective Length of Columns in Frames

4.6.1.1 In the absence of a more exact analysis, the effective length of columns in framed structures may be obtained by multiplying the actual length of the column between the centres of laterally supporting members (beams) with the effective length factor *K*, calculated by using the equations given below, provided the connection between beam and column is rigid type.

a) Non-sway Frames (Braced Frame)

A frame is designated as non-sway frame if the relative displacement between the two adjacent floors is restrained by bracings or shear walls. The effective length factor, *K*, of column in non-sway frames is given by

$$K = \frac{\left[1 + 0.145(\beta_1 + \beta_2) - 0.265\beta_1\beta_2\right]}{\left[2 - 0.364(\beta_1 + \beta_2) - 0.247\beta_1\beta_2\right]}$$

b) Sway frames (Moment Resisting Frames)

The effective length factor, *K*, of column in sway frames is given by

$$K = \left[\frac{1 - 0.2(\beta_1 + \beta_2) - 0.12\beta_1\beta_2}{1 - 0.8(\beta_1 + \beta_2) + 0.6\beta_1\beta_2}\right]^{0.5}$$

where

β_1, β_2 are given, $\beta = \dfrac{\sum K_c}{\sum K_c + \sum K_b}$

K_c, K_b = effective flexural stiffness of the columns and beams meeting at the joint at the ends of the columns and rigidly connected at the joints, and those are calculated by

$K = C\,(I / L)$

I = moment of inertia of the member about an axis perpendicular to the plan of the frame

L = length of the member equal to centre to centre distance of the intersecting member

C = correction factor as shown in Table 4.6.1

TABLE 4.6.1

Far end condition	Correction Factor C	
	Braced frame	**Unbraced frame**
Pinned	$1.5(1-\bar{n})$	$1.5(1-\bar{n})$
Rigidly connected to column	$1.0(1-\bar{n})$	$1.0(1-0.2\bar{n})$
Fixed	$2.0(1-0.4\bar{n})$	$0.67(1-0.4\bar{n})$

Note: $\bar{n} = \dfrac{P}{P_e}$

where P_e = elastic buckling load

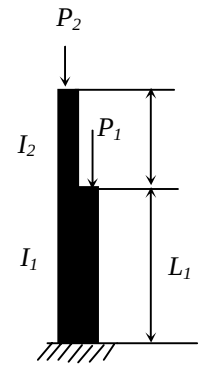
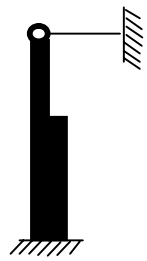
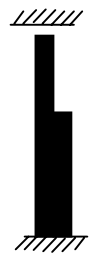
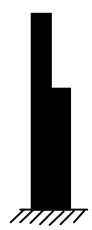
P = applied load

4.6.2 Method for determining Effective Length for Stepped Columns

4.6.2.1 Single Stepped Columns – Effective length in the plane of steeping (bending about axis *z-z*) for bottom and top parts for single stepped column shall be taken as given in Table E.2 of IS: 800.

TABLE E.2 EFFECTIVE LENGTH OF SINGLED STEPPED COLUMNS

(Appendix E.1.1)

Sl. No.	Degree of End Restraint	Sketch	Effective Length Coefficients	Column Parameters for all Cases
a)	Effectively held in position and restrained against rotation at both ends		$K_1 = \sqrt{\frac{K_{12}^2 + K_{11}^2(\alpha - 1)}{\alpha}}$ $K_2 = \frac{K_1}{C_1} \le 3$ where K_{12} and K_{11} are to be taken as per Table E.3 of IS: 800	 $\frac{I_2}{I_2 + I_1} = \alpha$ $C_1 = \frac{L_2}{L_1}\sqrt{\frac{I_1}{I_2\alpha}}$ $\frac{i_2}{i_1} = \frac{I_2}{L_2} \times \frac{L_1}{I_1}$ Effective length of bottom part of column in plane of stepping = K_1L_1 Effective length of top part of column in plane of stepping = K_2L_2
b)	Effectively held in position at both ends and restrained against rotation at bottom end only		$K_1 = \sqrt{\frac{K_{12}^2 + K_{11}^2(\alpha - 1)}{\alpha}}$ $K_2 = \frac{K_1}{C_1} \le 3$ where K_{12} and K_{11} are to be taken as per Table E.4 of IS: 800	
c)	Effectively held in position and restrained against rotation at bottom end, and top end held against rotation but not held in position		K_1 to be taken as per Table E.5 of IS: 800 $K_2 = \frac{K_1}{C_1} \le 3$	
d)	Effectively held in position and restrained against rotation at bottom end, and top end neither held against rotation nor held in position		K_1 to be taken as per Table E.6 of IS: 800 $K_2 = \frac{K_1}{C_1} \le 3$	

5.0 A SIMPLIFIED SWAY METHOD

This is one of the approximate methods recommended by BS 5950 for elastic design of sway frames. In this method, the effect of instability of the column on bending moments and deflection is considered by appropriately increasing their magnitude (magnifying) by a factor $\frac{1}{1 - \frac{P}{P_{cr}}}$ where P is the current load level and P_{cr} is the load required to cause

instability. This method has been tested for different ratios of moments acting at top and bottom of the column. If we designate this moment ratio as m_o (smaller end moment / larger end moment) the magnification factor due to instability for different ratios of m_o is shown (by Wood) as in Fig.10.

If $\lambda_{cr} = P_{cr} / P$ design, then the amplification factor will be

$$MF = \left(\frac{1}{1 - \frac{1}{\lambda_{cr}}} \right) = \left(\frac{\lambda_{cr}}{\lambda_{cr} - 1} \right) \qquad (3)$$

The influence of frame instability on elastic response is shown in Fig.11. BS 5950 in the simplified sway method requires that all moments obtained by elastic analysis due to horizontal forces be increased by this magnification factor. Since the effects of instability are incorporated by moment magnifier method, the effective length of the column is kept as actual length of the column itself.

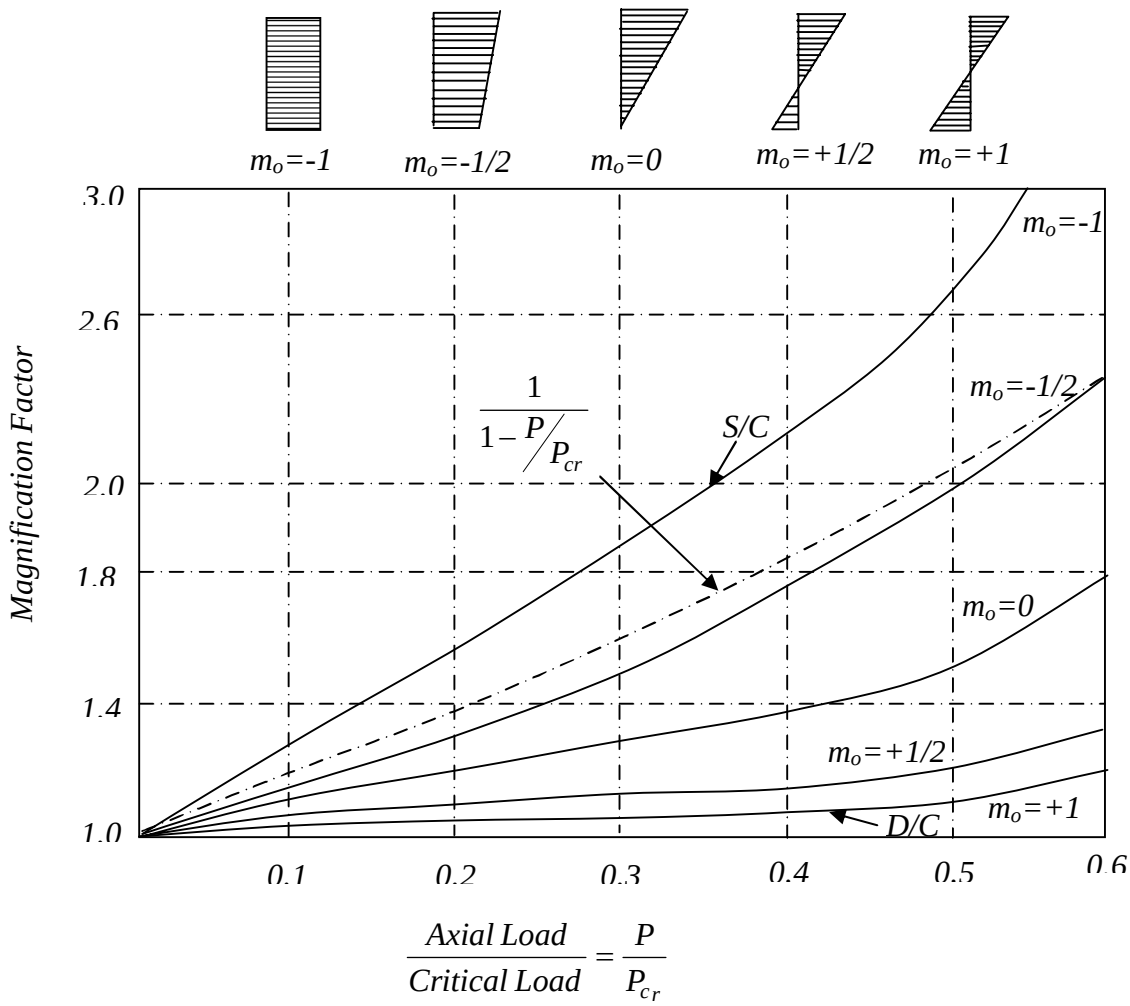


Fig. 10 Magnification of Moment due to axial load (non- sway)

S/C - Single curvature bending
D/C - Double curvature bending

$$m_o = \frac{Smaller\ end\ moment}{L\,arg\,er\ end\ moment}$$

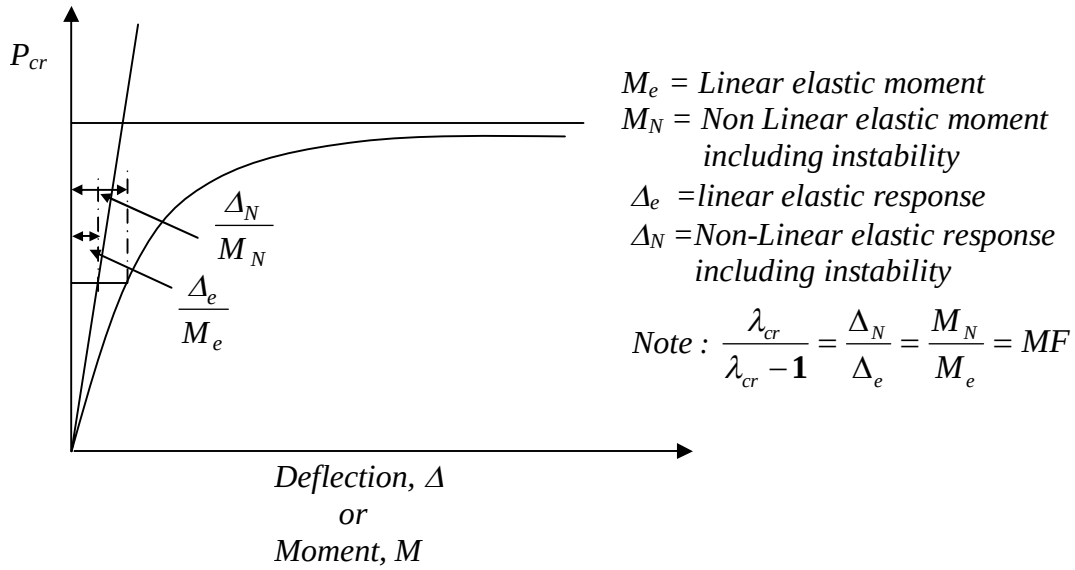


Fig. 11 Response (magnified) due to elastic instability

6.0 ELASTIC DESIGN OF MULTISTOREY RIGID FRAMES

6.1General

The elastic design as per BS 5950 is made for factored loads when the deflections are small. The deflections should generally be limited to *span/200*. In these cases, deflections do not cause any significant instability. The design of beams and columns are made using substitute frames for gravity loading described earlier. For horizontal loading it is necessary to consider entire frame. One of the approximate methods described earlier can be used. Even when elastic design is used, moment redistribution to the extent of *10%* can be made provided compact or plastic sections are used and minor axis column moments are not reduced while maintaining equilibrium.

6.2Non - sway frames

For gravity loading non-sway frames are analysed either using full frame or using substitute frame. The effective length of columns is obtained as described earlier in Section 4.0 taking them as braced. For load cases involving horizontal load, pattern vertical loading is not considered and the entire frame is analysed.

6.3Sway Frames

The frames, which exceed the non-sway limit as specified in Section 2.0 are designed considering sway.

As a first step, the frame is analysed for vertical gravity loading considering also pattern loading as a non-sway frame using effective length of columns applicable to those braced against sidesway.
Next, the effects of sway is considered under all combination of loading, considering vertical loading effects on sway, the notional lateral load as described in Section 2.0 is

applied at each storey level and one of the following two design methods is adopted to get the final design forces.

(i) Simplified Design Method

The side sway is allowed. The effective length as explained in Section 4.0 using limited frame method is used and the design forces are obtained.

(ii) Amplified Sway Method

The bending moments due to lateral loads are magnified by moment magnification factor $\left(\frac{\lambda_{cr}}{\lambda_{cr}-1}\right)$ as explained in Section 5.0 and the final design forces are obtained. Since the moments have been magnified the effective length of the column is assumed as actual length of column (i.e. $\ell_{eff} = \ell$).

7.0 STABILITY CONSIDERATIONS OF SWAY FRAME UNDER ELASTIC-PLASTIC FAILURE LOADS.

7.1Elastic Critical Conditions

In a normal elastic frame, the deflection function $F(\Delta)$ of the frame is proportional to the deflection $f(\Delta)$ of the frame under unit load. Thus

$$F(\Delta) = \lambda f(\Delta) \tag{4}$$

where λ is the load factor.

The axial forces in the column are proportionate to applied loading. These axial forces introduce the instability effects. It is necessary to compute the reduction in stiffness of the columns as they approach the critical loads. At certain critical load factors $\lambda_{c1} \leq \lambda_{c2} \leq \lambda_{c3}$ (the eigen values), the stiffnesses vanish leading to large deflections. These correspond to critical modes at those load factors. Eigen vectors at the corresponding loads are represented by deflection function $f(\Delta_1)$, $f(\Delta_2)$, $f(\Delta_3)$ etc.

Using the orthogonal property of the mode shapes, we can express the deflection as

$$f(\Delta) = a_1 f(\Delta_1) + a_2 f(\Delta_2) + a_3 f(\Delta_3) + \ldots \tag{5}$$

where a_1, a_2, and a_3 are participation factors for each mode.

When instability effects are considered the resulting deflection accommodating non-linear effect can be expressed as

$$f(\Delta) = \left(\frac{\lambda_{c1}}{\lambda_{c1} - \lambda}\right) a_1 \, f(\Delta_1) + \left(\frac{\lambda_{c2}}{\lambda_{c2} - \lambda}\right) a_2 \, f(\Delta_2) + \left(\frac{\lambda_{c3}}{\lambda_{c3} - \lambda}\right) a_3 \, f(\Delta_3) + \ldots\ldots\ldots\ldots \qquad (6)$$

Deflections approach very large values as λ approaches λ_{c1}, λ_{c2} etc. When deflections become large, it is not acceptable to express them in terms of eigen vectors and the deflection pattern will change the member forces in the columns. For deflections within practical limits, equation (6) is applicable.

It is necessary to find the lowest critical load because it shows the onset of elastic critical condition. The elastic critical load factor λ_{cr} of the frame is the ratio by which each of the factored loads will have to be increased to cause elastic instability.

This load factor is also required to be used in the approximate method for evaluating elastic-plastic failure loads. The deflection method given in Appendix F of BS 5950 Part 1: 1985 is an approximate method based on the work of Horne to arrive at a reasonable estimate of elastic buckling load λ_{cr}. This method is described below:

Consider the rigid frame shown in Fig.1 (*a*) and the analysis performed as indicated in Section 2.0 under lateral loads whose magnitude is *0.5%* of the factored dead and live loads as shown in Fig. 1 (*b*). The sway index of the typical i^{th} storey is

$$\phi_i = \frac{s_i}{h_i} \qquad (7)$$

Note that s_i is the ith storey inter storey displacement. Thus the values of $\phi_1, \phi_2, \phi_3, \ldots\ldots \phi_i \ldots\ldots \phi_n$ for all storeys are computed. If ϕ_{max} is the maximum of all ϕ_i values, then the elastic critical load factor is

$$\lambda_{cr} = \frac{1}{200\,\phi_{max}} \qquad (8)$$

Horne has shown that the above expression gives an approximate lower bound to the elastic critical load.

7.2 Deteriorated Critical Load

The stability of a structure depends on the equilibrium state with reference to the potential energy *U*. A structure with small deformation will have a typical load–deflection curve as indicated by curve *XYZ* in Fig.12 (*b*). The effect of load due to lateral deflection in these structures are not significant. The points *X*, *Y* and *Z* represent three different states of stability of the frame shown in Fig.12 (*a*). The potential energy *U* is the sum of the potential energy of loads U_w and the elastic strain energy stored U_e.

Thus

$$U = U_w + U_e \qquad (9)$$

The condition of stability of the frame can be assessed based on whether the first partial derivative with respect to deflection is greater than zero, less than zero or equal to zero. When it is greater than zero the system is stable. When it is equal to zero the system is neutral i.e. more displacement will not change the system. When it is less than zero the system is unstable i.e. a small change will cause collapse.

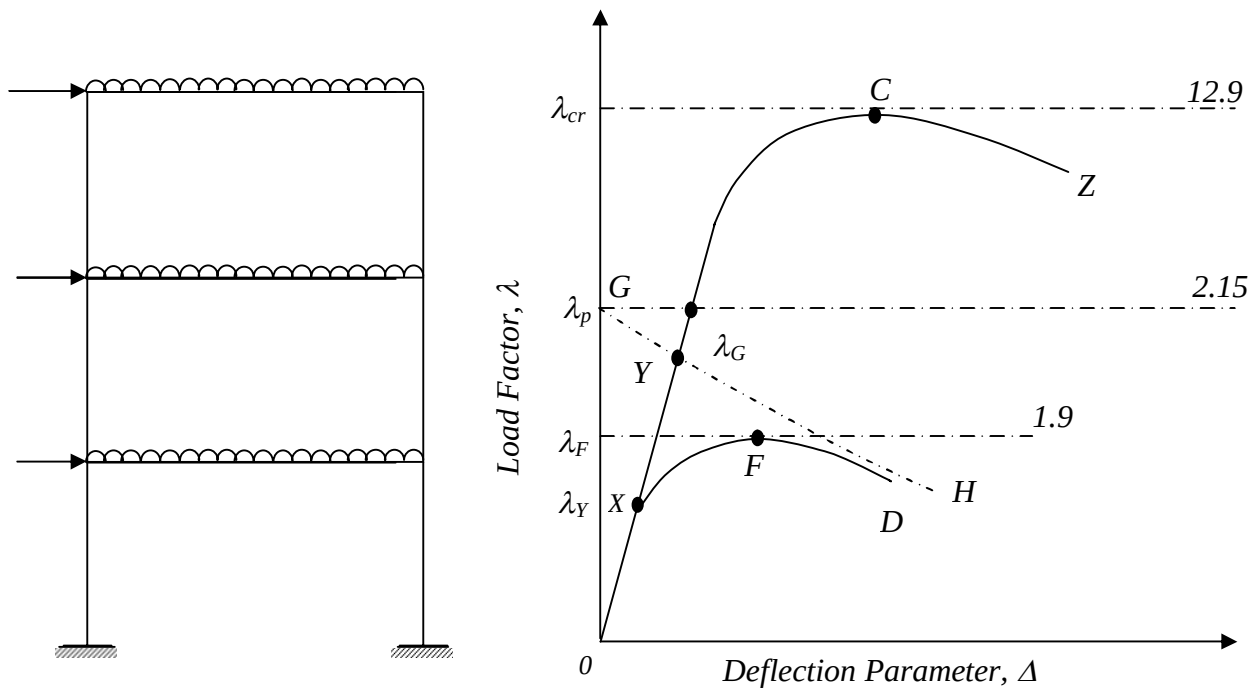
For equilibrium

$$\frac{\partial U}{\partial \Delta} = 0 \tag{10}$$

On the rising part, i.e., at point *X*,

$$\frac{\partial^2 U}{\partial \Delta^2} > 0 \ (Stable) \tag{11}$$

On the falling part, i.e. at *Z*



(a) *(b)*

Fig.12 Load-deflection curve for an elastic-plastic structure compared with an elastic structure

$$\frac{\partial^2 U}{\partial \Delta^2} < 0 \ (Unstable) \tag{12}$$

and at *C*

$$\frac{\partial^2 U}{\partial \Delta^2} = 0 \ (neutral) \tag{13}$$

The above explanation is valid for an elastic system undergoing instability problem.

Consider the load deflection curve *OXFD* in Fig.12 (*b*) for a typical elastic-plastic non-linear structure system. This should include U_p the energy absorbed in plastic deformation. Now the total energy U_N is

$$U_N = U_w + U_e + U_p \tag{14}$$

For equilibrium

$$\frac{\partial U_N}{\partial \Delta} = \mathbf{0} \ \text{ Valid for all points on OXFD} \tag{15}$$

In the plastic zone $\frac{\partial^2 U_p}{\partial \Delta^2} = 0$ since the stress is constant

$$\frac{\partial^2 U_N}{\partial \Delta^2} > 0 \quad \text{upto } \lambda_F \text{ is reached} \tag{16}$$

$$\frac{\partial^2 U_N}{\partial \Delta^2} = 0 \quad \text{at } \lambda_F \text{ i.e at point } F \tag{17}$$

$$\frac{\partial^2 U_N}{\partial \Delta^2} < \mathbf{0} \ \text{ beyond } F \text{ in the falling branch of } FD \tag{18}$$

The condition at failure

$$\frac{\partial^2 \left(U_w + V_e\right)}{\partial U^2} = 0 \tag{19}$$

From the above it is clear that failure criteria for elastic-plastic structure is similar to elastic structure with plastically deforming parts eliminated i.e. the plastically deforming parts contribution becomes zero. The elastic portion between plastic hinges will still be contributing to the energy. The structure with the eliminated parts is termed “deteriorated or depleted”. The critical load obtained under this depleted or deteriorated structure is known as deteriorated critical load.

The curve *OXC* represents the behaviour of ideally elastic frame.

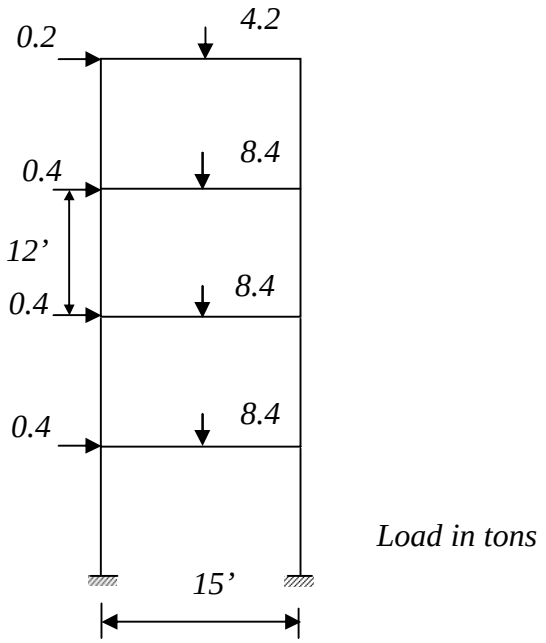


Fig.13 Frame analysed by Wood

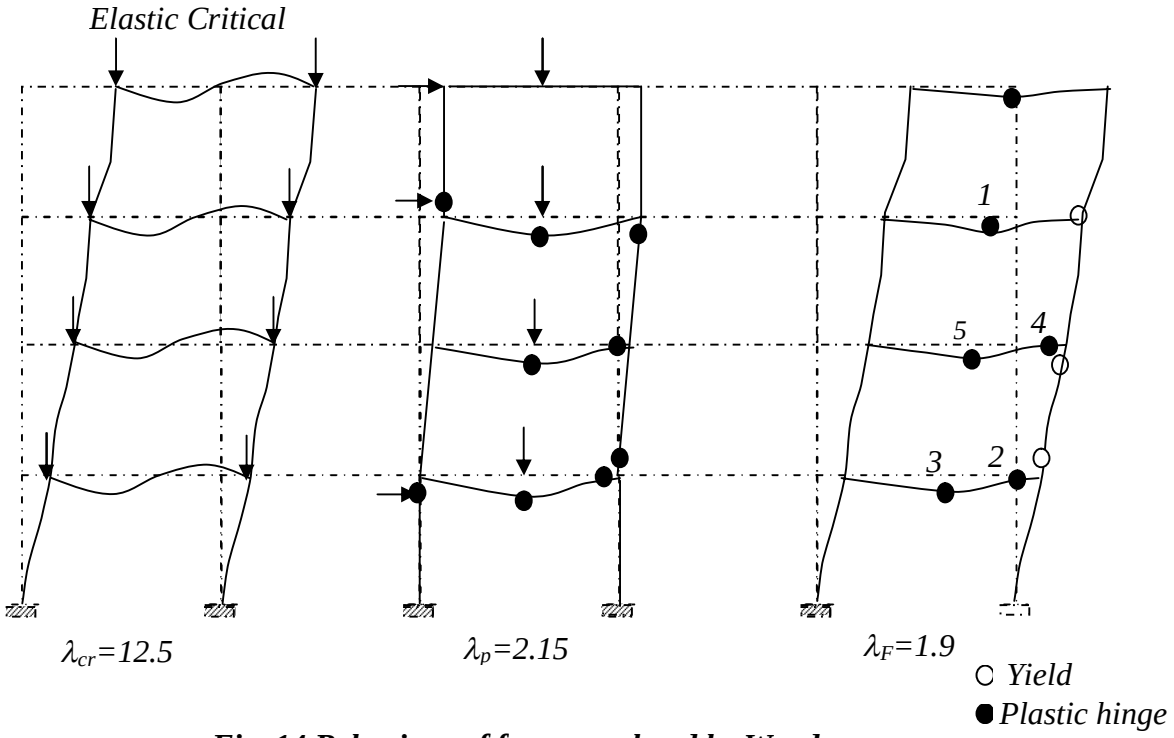


Fig. 14 Behaviour of frame analysed by Wood

The following are identified with respect to "deteriorated" critical load condition.

λ_{cr} = elastic critical load factor
λ_p = rigid plastic critical load factor
λ_G = rigid plastic critical load considering members between hinges formed.
λ_F = deteriorated critical load factor without the energy component of these parts which are plastically deforming
λ_y = load factor at on set of yield.

Wood analysed the frame shown in Fig.13. The values he obtained λ_{cr} , λ_p and λ_f the corresponding behaviour of the example frame is reproduced in Fig.14. The deterioration of critical load λ_D for the partially plastified structure is shown in Fig.15. None of the deteriorated structures correspond exactly to the actual structure with hinges at failure [Fig.14(*c*)] λ_F obtained at elastic plastic failure is *1.90*. Though ten hinges are required for the rigid plastic collapse of the frame, nearly two hinges are sufficient to reduce the elastic critical load by nearly *50%*.

Such a complete analysis as discussed above is required for a realistic estimate of deteriorated critical load. In the absence of sophisticated Computer Programme to carry out such an analysis, a simplified method is required for considering the deteriorated critical load for use by designers. Such an empirical approach proposed by Merchant Rankine Wood Equation is discussed in the next section.

8.0 SIMPLIFIED EMPIRICAL APPROACH USING MERCHANT – RANKINE – WOOD EQUATION

An examination of Fig.12 reveals that λ_{cr}, the elastic critical value is too high and cannot be reached. If rigid plastic behaviour is assumed the critical load is represented by the drooping curve *GH* descending from λ_p the rigid plastic load factor. Merchant suggested that realistic failure load λ_F can be expressed as a function of λ_p and λ_{cr}. According to original Merchant Rankine Equation.

$$\frac{1}{\lambda_F} = \frac{1}{\lambda_p} + \frac{1}{\lambda_{cr}} \tag{20}$$

This is shown in Fig.16 in which $\frac{\lambda_F}{\lambda_p}$ is plotted vertically against $\frac{\lambda_{MR}}{\lambda_{cr}}$. If we call the failure load as Merchant – Rankine load λ_{MR} then

$$\lambda_{MR} = \frac{\lambda_p \lambda_{cr}}{\lambda_p + \lambda_{cr}} \tag{21}$$

Wood suggested a modification of Merchant Rankine load considering strain- hardening and restraint provided by cladding

if

$$\frac{\lambda_{cr}}{\lambda_p} > 10 \text{ then } \lambda_F = \lambda_p \tag{22}$$

and

$$\begin{aligned}\lambda_F &= \lambda_{MRW} \\ &= \frac{\lambda_p \lambda_{cr}}{\lambda_p + 0.9\lambda_{cr}} \\ &= \frac{\lambda_p}{0.9 + \frac{\lambda_p}{\lambda_{cr}}}\end{aligned} \tag{23}$$

when $10 > \frac{\lambda_{cr}}{\lambda_p} > 4$; then

Consider stocky structures i.e.

with $\frac{\lambda_{cr}}{\lambda_p} > 10$ or $\lambda_{cr} > 10$ and $\lambda_p > 1$

ensures that structures have adequate strength. For slender structures

$$10 > \frac{\lambda_{cr}}{\lambda_p} > 4 \tag{24}$$

or

$4.6 < \lambda_{cr} < 10$ as used in BS 5950 the values of $\lambda_p \geq \frac{0.9\lambda_{cr}}{(\lambda_{cr} - 1)}$

and $\lambda_{cr} > 4.6$ (25)

This is applicable to clad frames in which no account has been taken of cladding.

These equations are modified for unclad frames or frames where stiffness of cladding is considered as indicated below:

$\lambda_{cr} \geq 5.75$ or (26)

$5.75 \leq \lambda_{cr} < 20$ (27)

$$\lambda_p \geq \frac{9.5\,\lambda_{cr}}{(\lambda_{cr} - 1)} \tag{28}$$

when

$\lambda_{cr} \geq 20$; use $\lambda_p \geq 1$ (30)

Thus the method involves finding λ_{cr}, the elastic critical load and λ_p the rigid plastic critical load and then appropriate equation satisfied based on whether the frame is a clad one or otherwise.

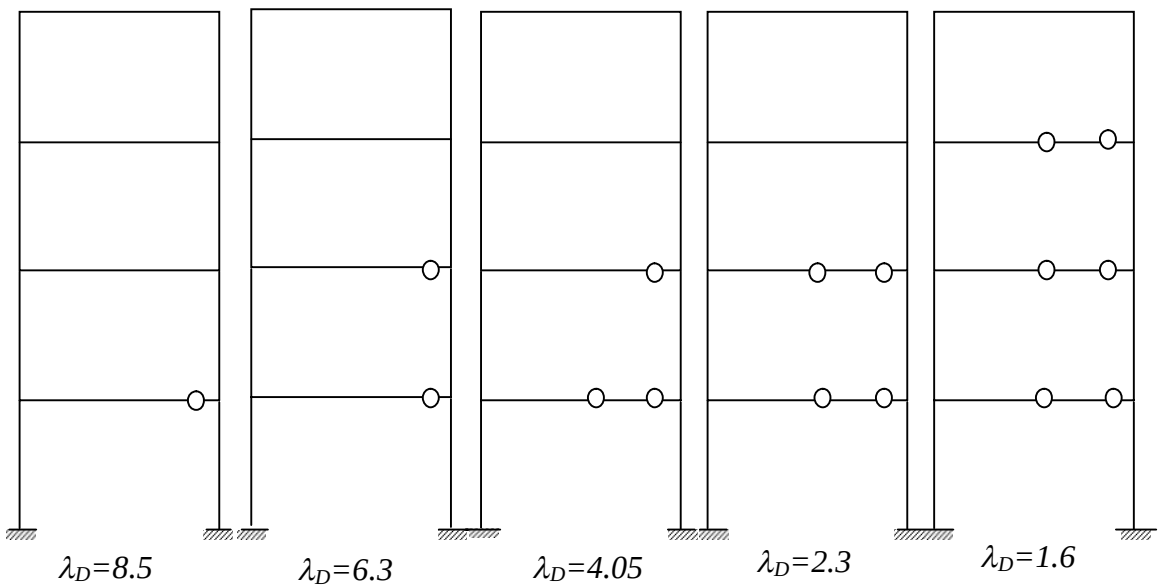


Fig. 15 Deteriorated Critical Loads of Frame analysed by Wood

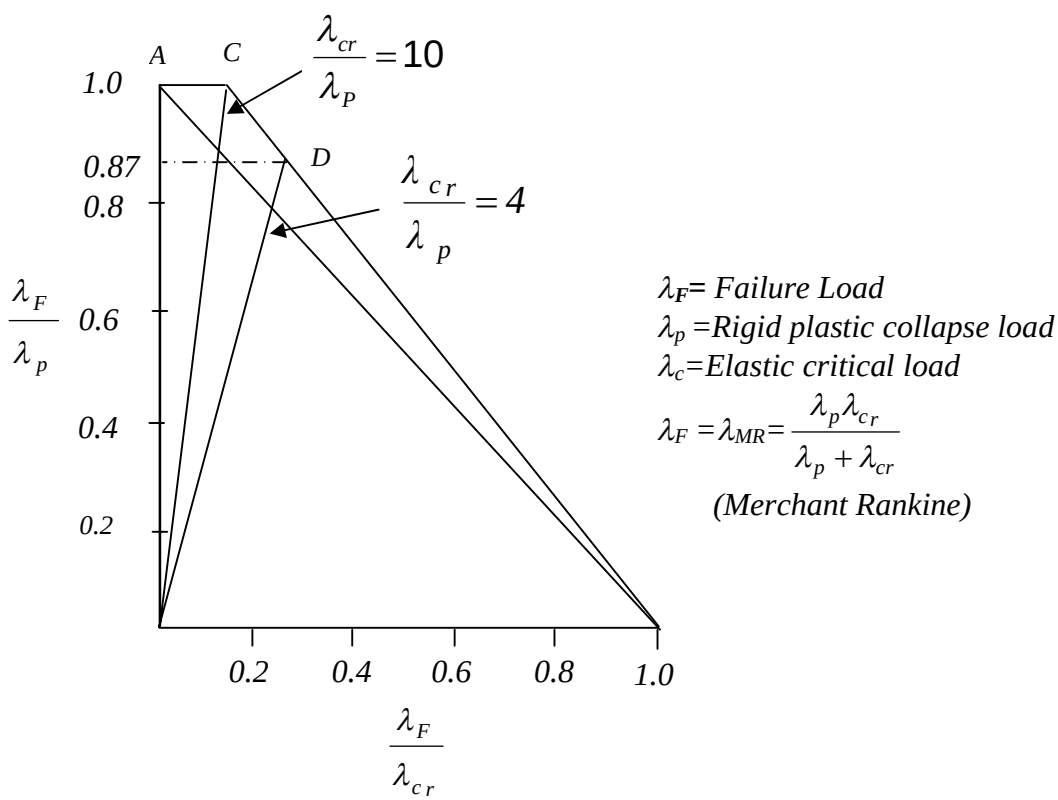


Fig. 16 Merchant - Rankine (modified Load)

9.0 PLASTIC DESIGN OF MULTISTOREY RIGID FRAMES

Plastic design of frames can be used for frames, which are effectively braced against out of plane sway.

9.1 Non Sway Frame

The frame should be braced against lateral sway such that it can be classified as a non-sway frame as per the condition explained in Section 2.0. However, while considering the sway, against lateral loads, the bending stiffness of the frame should be ignored, as its buckling resistance will not be available to prevent sidesway when the frame reaches its plastic capacity.

9.2 Sway Frames

Either of the following two methods is used:

a) Rigorous Analysis: A full elastic-plastic sway analysis is performed where proper allowance is made for frame instability effects as indicated in Section 7.2.

b) Simplified Empirical Approach: A simplified frame stability check, as given in Section 7.3, is made using Merchant–Rankine–Wood Equation provided the following conditions are satisfied.

   (i) The beam side-sway mechanism with hinges in all beam ends and at base of columns should be applicable. There should not be other hinges in the column, which may lead to premature failure.
   (ii) The column in the ground floor should be designed to remain within elastic limit.
   (iii) Under the combination of unfactored load and notional horizontal load to simulate sway (wind force not included), forces and moments in the frame should be within elastic limit.

10.0 SUMMARY

In this chapter the behaviour of multistorey frames under lateral loads is described. Elastic design of multistorey rigid frames using simplified design method as well as amplified sway method have been included. Stability consideration of sway frames under Elastic-Plastic failure loads have been included. Finally plastic design of multistorey frame using simplified approach has been is presented.

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